Location-based Joint Relay Selection and Channel Allocation for Cognitive Radio Networks*

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Abstract—In cognitive radio networks (CRNs), dynamic spectrum access has been demonstrated as an effective way to improve the spectrum utilization. Spectrum holes can be exploited not only in certain time slots or frequency bands, but also at particular locations. In relay assisted CRNs, one relay at a certain location can help to identify and provide different spectrum holes over multiple channels. In this paper, a multi-dimensional combinatorial optimization problem is formulated for joint relay selection and channel allocation. We propose a weighted bipartite graph model and a minimum weighted assignment approach to efficiently get the optimal solution of the considered problem. Simulation results show that by applying this approach, spectrum efficiency, relay selection diversity and power efficiency can be improved simultaneously for the cognitive users. Besides, only the statistical channel state information is needed and the allocation results can be computed efficiently by using the proposed approach.

I. INTRODUCTION

COGNITIVE radio has been considered as a key enabling solution to the spectrum scarcity problem by applying dynamic spectrum sharing [1]. It allows cognitive users to share the spectrum owned by the licensed users in the primary network (PN) without introducing much interference. In cognitive radio networks (CRNs), the key challenge lies in how to improve the spectrum efficiency by taking advantage of the multi-dimensional spectrum holes. Most of the previous works on cognitive radio focused on the sensing and utilization of spectrum holes in the frequency or time domain, while how to improve the utilization of spectrum holes by using the location information of the primary and cognitive users has not been studied in a systematic way. With path loss, the location plays a key role to leverage the interference between primary and cognitive users. Location awareness can help find multiple local spectrum holes and a cognitive user may be encouraged to access the frequency band owned by distant primary users. The location information can be obtained by the positioning system GPS or other localization techniques [2].

There are only a few related works on location based spectrum access. Zheng and Peng considered the differences in the spectrum environment for cognitive users in different locations [3]. A graph coloring method was then proposed to deal with the spectrum allocation in that system. Bai et al. proposed a location-based cluster and list-coloring method for spectrum sharing [4]. A distributed coordination scheme for dynamic spectrum allocation that takes users’ location into account was proposed in [5]. Xie et al. introduced a geometric routing architecture to improve the spectrum efficiency [6]. In [7], an infrastructure based CRN was proposed which uses cooperative relays to improve the spectrum efficiency. A heuristic greedy algorithm was developed for the relay selection and channel allocation.

In the CRNs, cognitive relays can help to reduce the interference and acquire more spectrum holes. Cognitive users can usually work over multiple channels and the channels may be occupied by the primary users at different positions. In addition, one cognitive relay at a certain location may find different spectrum holes over different channels so that it can help multiple cognitive users. Therefore, for cognitive users, relay selection and channel allocation can be jointly optimized to achieve high spectrum efficiency and relay selection diversity. It should be noticed that power control is a key factor for the joint optimization, because it not only controls the interference between the primary and cognitive users, but also can improve the power efficiency.

In this paper, we consider a cognitive radio system with multiple source-destination pairs who want to share multiple channels licensed to the primary pairs. Multiple cognitive relays are deployed to assist the cognitive pairs to achieve successful communications. We consider the cross-layer design for joint relay selection and channel allocation with two objectives: 1) to maximize the number of active cognitive pairs, and 2) to minimize the sum transmission power of the CRN. A two-stage two-objective combinatorial optimization problem is formulated for joint resource allocation. Such a problem is difficult and of high computation complexity. Observing the special structure of our problem, we propose a weighted bipartite graph model to reformulate it. Based upon the weighted bipartite graph, a minimum weighted assignment method is developed to perform the joint relay selection and channel allocation, with the two objectives achieved simultaneously. Compared with the heuristic greedy algorithm, the simulation results show that more cognitive pairs can be active when employing the proposed method, which implies a higher spectrum efficiency.

The rest of the paper is organized as follows. The system model is given in Section II and then the problem formulation is presented in Section III. A weighted bipartite assignment method is proposed for the joint relay selection and channel allocation in Section IV. Some numerical results which illustrate the potential of the proposed approach are shown in Section V. Finally, Section VI concludes the paper.

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Figure 1. System model of the relay-assisted CRN coexisting with the PN: \( M \) primary source-destination (PS,PD), \( N \) cognitive source-destination (CS,CD), \( K \) cognitive relays (CR) and \( M \) channels (Ch).

II. SYSTEM MODEL

Consider a multi-channel spectrum sharing CRN, as shown in Fig. 1, which is composed of \( N \) source-destination pairs. The direct link is not considered, since a longer transmission range introduces higher interference to primary users. \( K \) cognitive relays are deployed to assist the communications of the CRN. In the same region, there are \( M \) primary source-destination pairs occupying \( M \) orthogonal channels. The cognitive users can access the channels only if they do not cause high interference to primary users. We assume that the locations of all the users in the CRN and PN are known to the cognitive users by applying some positioning algorithms [2].

As shown in Fig. 1, \( N \) cognitive pairs are denoted as \( \{(CS_m, CD_n)\}_{n=1,2,...,N} \) and \( K \) cognitive relays are denoted as \( \{CR_k\}_{k=1,2,...,K} \). \( M \) licensed channels \( \{Ch_m\}_{m=1,2,...,M} \) are used by \( M \) primary pairs \( \{PS_m, PD_m\}_{m=1,2,...,M} \) respectively. Let \( d_{AB} \) denote the distance between the two subscript nodes.

The channels between any transmission/interference source and receiver are assumed to be flat Rayleigh fading and the outage probability is mainly affected by the signal-to-interference-plus-noise-ratio (SINR) with large-scale fading. Due to the hardware limitation, the cognitive relays are as-sumed to work in a half-duplex and decode-and-forward mode. All the channels are orthogonal and occupied by the primary pairs all the time. For channel allocation, we assume that the 1st and 2nd hops for one cognitive pair use the same channel. One channel can be used by only one pair so that there is no interference between the cognitive pairs. For relay selection, one relay may be used by multiple pairs as needed. The interference exists between and only between the cognitive and primary pairs who share the same channel, at both the 1st and 2nd hops.

Taking advantage of the location information, we first consider to achieve the objective of maximizing the number of the active cognitive pairs by joint relay selection and channel allocation. We also set minimizing the total transmission power of the CRN as the second objective, which can save power consumption and reduce the interference between the CRN and other networks.

III. JOINT RELAY SELECTION AND CHANNEL ALLOCATION

In the considered system, there are multiple relays and multiple channels that may be used by the cognitive pairs. To achieve the designed objectives, the relay selection and channel allocation are needed and they are coupled. For this joint relay selection and channel allocation problem, we define a joint allocation matrix \( \Phi = [\phi_{mnk}]_{M \times N \times K} \) as

\[
\phi_{mnk} = \begin{cases} 
1, & \text{Ch}_m, \text{CR}_k \text{ are allocated to } (CS_n, CD_n), \\
0, & \text{otherwise}.
\end{cases}
\]

A. Outage Probability and Power Control

The quality-of-service (QoS) of both the primary users and cognitive users should be guaranteed so that they can work properly. In the Rayleigh fading scenario, power control is used to guarantee the QoS in terms of the outage probability (the source-destination pair is in outage when the pre-defined transmit rate \( R \) is not achieved). The mutual information \( I \) is given by \( I = \log (1 + \text{SNR}) \). The outage probability, \( \xi^\text{out} \), based on the statistical channel state information in Rayleigh fading network is given by [8]:

\[
\xi^\text{out} = \Pr \{ I < R \} = \exp \left( -\frac{\Theta \sigma_0^2}{\sigma_0^2} \right) \times \frac{1}{1 + \Theta \frac{P_t d_{t}^{-\alpha}}{P_i d_i^{-\alpha}}},
\]

where \( P_t \) and \( P_i \) are the transmission power of the transmission and interference source respectively, \( \sigma_0^2 \) is the variance of the channel without large-scale fading, \( \sigma_n^2 \) is the noise power, \( d_t \) and \( d_i \) are the distances between the transmission, interference sources and the receiver respectively, \( \alpha \) denotes the path-loss exponent and \( \Theta = 2^R - 1 \). We see that the signal-to-noise-ratio (SNR) and signal-to-interference-ratio (SIR) effects are independent, and the transmission power and distance are the main impact factors.

The transmission power of the cognitive users should not be too large to interfere with the primary users. The transmission power of the primary sources, which is denoted by \( P_{PS} \), are assumed to be the same to simplify the formulation. Thus, \( \mu_{\text{SNR}} \) for the primary pairs is a constant. When \( \phi_{mnk} = 1 \), to make sure that the outage probability of the primary pair \( (PS_m, PD_m) \) is lower than a pre-defined threshold \( \epsilon_0 \), the transmission power of the specific cognitive source and relay, denoted as \( P_{\text{CS mnk}} \) and \( P_{\text{CR mnk}} \), should satisfy [6]

\[
\begin{align*}
P_{\text{CS mnk}} & \leq \mu_{\text{SNR}}^{\epsilon_{0}} \left( 1 - \epsilon_{0} \right) \frac{d_{PS_m, PD_m}}{d_{CS_m, CD_n}} P_{PS} \leq P_{\text{CS max}}^{\epsilon_{0}} \quad (2) \\
P_{\text{CR mnk}} & \leq \mu_{\text{SIR}}^{\epsilon_{0}} \left( 1 - \epsilon_{0} \right) \frac{d_{CR_k, CD_n}}{d_{CS_m, CD_n}} P_{PS} \leq P_{\text{CR max}}^{\epsilon_{0}}.
\end{align*}
\]

Meanwhile, the transmission power of the cognitive users should not be too small to have their own transmission in
outage either. Proposition 1 is given to help the QoS analysis for the cognitive users.

**Proposition 1.** In a Rayleigh fading CRN, the outage probability is lower than a pre-defined threshold \( \Phi^c \) if the minimum transmission power, denoted as \( P_{t}^{\min} \), satisfies

\[
P_{t}^{\min} = \frac{\Theta^2 P_t^0}{W} \left( \frac{\sigma_n^2/\sigma_0^2}{(1-\eta_n)P_{t}^0 d_{1,0}^{\alpha}} \exp \left( \frac{\alpha}{P_{t}^0 \sigma_0^2 d_{1,0}^{\alpha}} \right) \right) - \frac{\sigma_n^2/\sigma_0^2}{P_{t}^0 d_{1,0}^{\alpha}} d_t^{\alpha},
\]

where \( W(x) \) is the Lambert W-function [9].

**Proof:** The proof is omitted due to the space limitation.

Thus, when \( \phi_{mnk} = 1 \), the transmission power of the specific cognitive source and relay should satisfy

\[
\begin{align*}
\frac{P_{CR}^{mnk}}{\phi_{mnk}} &\geq \frac{\Theta^2 \sigma_n^2/\sigma_0^2}{W} \left( \frac{\sigma_n^2/\sigma_0^2}{(1-\eta_n)P_{t}^0 d_{1,0}^{\alpha}} \exp \left( \frac{\alpha}{P_{t}^0 \sigma_0^2 d_{1,0}^{\alpha}} \right) \right) - \frac{\sigma_n^2/\sigma_0^2}{P_{t}^0 d_{1,0}^{\alpha}} d_t^{\alpha},
\end{align*}
\]

\[
\begin{align*}
\frac{P_{CS}^{mnk}}{\phi_{mnk}} &\geq \frac{\Theta^2 \sigma_n^2/\sigma_0^2}{W} \left( \frac{\sigma_n^2/\sigma_0^2}{(1-\eta_n)P_{t}^0 d_{1,0}^{\alpha}} \exp \left( \frac{\alpha}{P_{t}^0 \sigma_0^2 d_{1,0}^{\alpha}} \right) \right) - \frac{\sigma_n^2/\sigma_0^2}{P_{t}^0 d_{1,0}^{\alpha}} d_t^{\alpha},
\end{align*}
\]

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

Constraints (6) and (7) are the power control constraints integrated with the allocation matrix. Constraint (8) is the channel allocation constraint as mentioned before. Objective (5) in (P) should be solved at the first stage and then Objective (4) is needed to be solved based on the result of Objective (5). Meanwhile, the maximum number of the active cognitive pairs \( N_{\text{act}} = |\Phi^*| \). (P) is an integer linear programming with a variable of 3-dimensional matrix. There are few algorithms can get the optimal solution to the best of our knowledge. To solve Objective (4) in (P), we need to find the argument \( \Phi^* \) for the maximization, which is even tough. In the next section, we will propose a minimum weighted bipartite assignment method to solve two objectives at the same time efficiently.

### IV. Weighted Bipartite Assignment Method

The joint relay selection and channel allocation as formulated in (P) is a multi-dimensional combinatorial optimization problem [10]. It is known that such kind of problems are very difficult to solve or to get the global optimal solution. Based on the features of this problem, we will reformulate it as a weighted bipartite graph and develop a minimum weighted assignment method to perform the relay selection and channel allocation.

Recalling that one relay may serve multiple cognitive pairs with different channels, cognitive relays are not one-to-one corresponding to cognitive pairs or channels, i.e., there is no constraint \( \sum_{m,n} \phi_{mnk} \leq 1 \) for the joint allocation matrix \( \Phi \). Thus we see the opportunity to reduce the dimension of joint allocation matrix, from a 3-dimensional space to a 2-dimensional space.

A graph is bipartite if it has two classes of vertices and the edges are only allowed between vertices of different classes, which is denoted by \( G(V_1 \cup V_2, E) \) and there is an associated value \( w_e \) for edge \( e \in E \) in a weighted bipartite graph (WBG) [11]. We set \( V_1 = \{ (C_{n}, C_{d}) \}_{n=1,2,...,N} \) and \( V_2 = \{ C_{m} \}_{m=1,2,...,M} \). We also define a channel allocation matrix \( \Psi \) as

\[
\psi_{mn} = \begin{cases}
1, & \text{Ch}_m \text{ is allocated to } (C_{n}, C_{d}) , \\
0, & \text{otherwise},
\end{cases}
\]

and a set \( R_{mn} = \{ k \mid \text{Eq. (6) is satisfied} \} \) to represent the indices of all of the available cognitive relays which can help the cognitive transmission when \( w_{mn} = 1 \). For the edge set \( E \), there is an edge between \( (C_{n}, C_{d}) \) and \( \text{Ch}_m \) if and only if \( R_{mn} \neq \emptyset \). To minimize the total transmission power, the locally "best" relay should be the one who helps the two hops consume the least power. Therefore, with Eq. (3), the weight of each edge is given by

\[
w_{mn} = \min_{k \in R_{mn}} \{ P_{CS}^{mnk} + P_{CR}^{mnk} \}.
\]

The optimal solution to Eq. (11), i.e., the locally "best" relay, is denoted as \( k_{mn}^* \). So far, the WBG model is set and the dimension of the original problem is reduced.

Based on the WBG we have now, Problem (P) can be directly transformed to a general matching problem. Objective (5) corresponds to finding all the maximum cardinality matchings based on this bipartite graph. Then, the total transmission power of every matching should be calculated and compared to get the minimum one, which is corresponding to Objective (4). The computation complexity is \( O \left( \sqrt{NE} + N \cdot F(N, E) \right) \)
for enumerating all the maximum cardinality matchings [12] while \( O(A) \) for comparing the total transmission power, where \( E = |\mathcal{E}| \) and \( F(N, E) \) is the number of the maximum cardinality matchings of the WBG. Both steps would have exponentially increasing complexity.

To solve the problem efficiently, we develop a minimum weighted bipartite assignment algorithm based on the above WBG. An assignment is a matching based on a complete bipartite graph [11]. Thus we first complete the WBG by inserting missing edges with weight \( L \), a large enough constant, i.e., \( L \gg \sum_{m,n,k} (P_{\text{CS}mnk} + P_{\text{CR}mnk}) \). Thus, for the complete WBG, the new weight is given by

\[
\tilde{w}_{mn} = \begin{cases} 
    w_{mn}, & R_{mn} \neq \emptyset, \\
    L, & \text{otherwise};
\end{cases}
\]

Then, we perform a minimum weight assignment based on this complete WBG. In this assignment, the more \( \tilde{w}_{mn} \) with the smaller value is taken, the better solution is obtained, which is consistent with our two objectives. Note that in the solution, there may be edges with weight \( L \), as well as \( R_{mn} = \emptyset \). However, for this case, \( \text{Ch}_m \) can not be allocated to the pair \((\text{CS}_n, \text{CD}_n)\) because no cognitive relay can help the transmission. Thus, we need to delete such kind of edges from the minimum weighted assignment. As a result, the optimal relay selection and channel allocation is achieved. Algorithm 1 shows the detailed steps of the minimum weighted assignment method. Fig. 2 is an example of the minimum weighted assignment, in which the result are drawn by thick lines and all the edges with weight \( L \) has been removed. By applying the cost-scaling algorithm, the minimum weighted assignment can be obtained with the computation complexity of \( O \left( \sqrt{NE \log(NL)} \right) \) [11]. Compared with \( O \left( \sqrt{NE + N \cdot F(N, E)} \right) \), the exponentially increasing complexity by using the general matching method, the proposed method is much more efficient.

**Algorithm 1** Minimum weighted bipartite assignment method

1. Construct the complete WBG \( \mathcal{G}(\mathcal{V}_1 \cup \mathcal{V}_2, \mathcal{E}) \): \( \mathcal{V}_1 = \{(\text{CS}_n, \text{CD}_n)\}_{n=1,2,\ldots,N} \) and \( \mathcal{V}_2 = \{\text{Ch}_m\}_{m=1,2,\ldots,M} \) with

\[
\tilde{w}_{mn} = \begin{cases} 
    w_{mn}, & R_{mn} \neq \emptyset, \\
    L, & \text{otherwise};
\end{cases}
\]

2. Execute the minimum weighted assignment, i.e., applying the cost-scaling algorithm [11]. As a result, we get an assignment \( \mathcal{M} \) with objective value \( \tilde{w}_{\text{sum}} \).

3. Find out the edges with weight \( L \) (t edges) in the assignment \( \tilde{\mathcal{M}} \) and remove them. The remaining subset \( \mathcal{M} \) is the final assignment result, in which each edge connects a cognitive pair \((\text{CS}_n, \text{CD}_n)\) and a channel \( \text{Ch}_m \), and also associated the "best" cognitive relay \( \text{CR}_k \).

4. The maximum number of the active cognitive pairs is \( N_{\text{act}} = \min\{N, M\} - t \) and the minimum total transmission power is \( P_{\text{CRN}} = \sum_{e_{mn} \in \mathcal{M}} w_{mn} \).

**V. SIMULATION RESULTS**

In this section, we present some simulation results to demonstrate the performance of the proposed minimum weighted assignment method for joint relay selection and channel allocation. We will compare our method with the exhaustive search method and a greedy algorithm. By using the exhaustive search method, all the allocation solutions are enumerated and compared so as to get the best one, which gives the global optimal solution with exponentially increasing complexity. Another intuitive method to solve the problem is the greedy algorithm as follows. By checking the power control constraints, we can always find out whether there is any channel coupled with any relay that can be used by a specific cognitive pair. We greedily choose the pair with the least cost one by one. The greedy algorithm gives a suboptimal solution and the computation complexity of it is \( O \left( N^2 \right) \).

In the simulation, there are \( N = 4 \) cognitive pairs in the CRN and \( M = 4 \) primary pairs in the PN. At the initial state, we randomly put \( K = 8 \) cognitive relays in the middle of the cognitive pairs and the geographic locations of all the users are shown in Fig. 3. For all the transmissions, the target range is fixed as \( R = 2 \), and the path-loss factor is given by \( \alpha = 3 \). For convenience, the pre-defined thresholds of the outage...
probabilities of the primary and cognitive users are the same and fixed as \( \epsilon_0 = \zeta_0 = 0.1 \) [6]. \( P_{PS} \) will be increased since it is a key factor to affect the location-based outage probability. Then we will move the PN away from the CRN to see the impacts.

Figure 4 shows that the proposed algorithm can always achieve the global optimal solution as the exhaustive algorithm, when the transmission power of the primary users \( P_{PS} \) increases. This verifies the optimality of our algorithm. Compared with the greedy method, we can see: 1) when \( P_{PS} \) is low, as shown in Fig. 4(a), more cognitive pairs can be active by applying the proposed algorithm than the greedy algorithm, because the primary users can be easily interfered when their own transmission power is low. Taking \( P_{PS} = 5 \) as an example, the proposed algorithm makes 3 cognitive pairs communicate successfully while the greedy algorithm makes 2. 2) when \( P_{PS} \) is high, Fig. 4(a) shows the same number of cognitive pairs are active. Fig. 4(b), however, shows that the proposed algorithm requires much less transmission power, about 5 dBW, which implies a higher power efficiency.

In Fig. 3, the locations of the PN and CRN are fixed. Now we move the whole PN up, away from the CRN, i.e., the distance between the CRN and PN increases. Figure 5(a) shows that we can have more cognitive pairs active by applying the proposed algorithm when the CRN is close to the PN and all the cognitive pairs are active by any algorithm when the CRN is far away from the PN. It is also shown that when the same number of cognitive pairs work well, the proposed algorithm consumes less power. This demonstrates the importance of the node locations for the considered problem.

At the left hand side of the dash dot line in both Fig. 4(b) and Fig. 5(b), it shows that the greedy algorithm gets lower total transmission power. If we refer to the corresponding parts in Fig. 4(a) and Fig. 5(a), more active pairs are active by applying the proposed algorithm. Therefore, the greedy algorithm cannot achieve the objective of maximizing the number of active cognitive pairs.

VI. CONCLUSIONS

In this paper, we investigated the joint relay selection and channel allocation in a location based CRN over Rayleigh fading channels. It was formulated as a multi-dimensional combinatorial optimization problem with the objectives of maximizing the number of active cognitive pairs while minimizing the total transmission power. Power control was used to reduce the interference so that more spectrum holes can be exploited. A weighted assignment approach based on a weighted bipartite graph reformulation was proposed to solve the optimization problem with low complexity. The results showed the significant gain compared with a heuristic greedy algorithm. The proposed approach not only provides a systematic way to solve the joint relay selection and channel allocation problem, but also improves the spectrum efficiency, relay selection diversity, along with power efficiency.

![Figure 4. Joint relay selection and channel allocation when \( M = N = 4, K = 8, \alpha = 3, R = 2, \epsilon_0 = \zeta_0 = 0.1 \) and CRN and PN are close.](image)

![Figure 5. Joint relay selection and channel allocation when \( M = N = 4, K = 8, \alpha = 3, R = 2, \epsilon_0 = \zeta_0 = 0.1 \) and \( P_{PS} = 10 \) dBW.](image)

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