Group Sparse Beamforming for Green Cloud Radio Access Networks

Yuanming Shi, Jun Zhang, and Khaled B. Letaief, Fellow, IEEE
Dept. of ECE, The Hong Kong University of Science and Technology
E-mail: {yshiac, eezhang, eekhaled}@ust.hk

Abstract—A cloud radio access network (C-RAN) is a promising network architecture to meet the explosive growth of the mobile data traffic. In this architecture, all the baseband signal processing is shifted to a single baseband unit (BBU) pool, which enables efficient resource allocation and interference management. Meanwhile, conventional powerful base stations can be replaced by low-cost low-power remote radio heads (RRHs), producing a green and low-cost network. However, as all the RRHs need to be connected to the BBU through backhaul links, the backhaul power consumption becomes significant and cannot be ignored. In this paper, we propose a new framework to design green C-RAN. Instead of only focusing on the RRH power consumption, we will minimize the network power consumption which includes the power consumed by both the RRHs and the backhaul links. The design problem is formulated as a joint RRH selection and power minimization beamforming problem, which turns out to be a convex-cardinality optimization problem and is NP-hard. We will first propose a global optimization algorithm based on the branch-and-bound method. By inducing the group-sparsity of the beamformers, we then propose two low-complexity algorithms, which essentially decouple the RRH selection and the power minimization beamforming. Simulation results demonstrate that the proposed algorithms can significantly reduce the network power consumption.

I. INTRODUCTION

To meet the expected mobile data traffic explosion in next generation cellular networks, more and more access points (APs) will be deployed to exploit the spatial reuse through the use of a small cell network [1]. Meanwhile, as stated in [2], placing APs based on the traffic demand is an effective way for compensating path-loss, resulting in energy efficient cellular networks. However, efficient interference management is challenging for dense cellular networks. Moreover, deploying more and more APs will cause significant cost and challenges for operators. Cloud radio access network (C-RAN) has recently been proposed as a promising network architecture to implement small-cell networks to jointly manage the interference, increase network capacity, and reduce both the network CAPEX and OPEX [3], [4].

In the C-RAN, the distributed transmission/reception points, called remote radio heads (RRHs), are connected to the baseband unit (BBU) pool through high bandwidth backhaul links [3]. With the centralized BBU pool and supported by real-time cloud computing, C-RAN enables efficient cooperation of the transmission/reception among different RRHs. As a result, significant performance improvements through joint scheduling and joint signal processing such as coordinated beamforming or multi-cell processing [5] can be achieved. With efficient interference suppression, a high density RRHs based network can be deployed. This will reduce the communication distance to the mobile terminals and can thus significantly reduce the transmit power. Moreover, as baseband signal processing is shifted to the BBU pool, RRHs only need to support basic transmission/reception functionality, which further reduces their energy consumption.

Unfortunately, the new architecture of C-RAN also indicates a paradigm shift in the network design. Conventionally, the backhaul power consumption can be ignored as it is negligible compared to the power consumption of macro base stations (BSs). Therefore, all the previous works investigating the energy efficiency of cellular networks only consider BS power consumption [6], [7]. However, in C-RAN, the data transmitted between the RRHs and the BBU pool is typically oversampled I/Q streams in the order of Gbps. The power consumption of such high-capacity backhaul links is comparable to that of the RRHs. For example, the power consumption of the backhaul link with 1 Gbps capacity is about $5W$ [8], while the transmit power consumption at a typical RRH is about $1W$ [9] with an inefficiency coefficient $\rho = 0.2$. As a result, to investigate the energy efficiency of the C-RAN, it is essential to consider the power consumption at the architectural level instead of only considering the RRH power consumption.

In this paper, we will investigate the network power minimization problem in C-RAN. This design problem is formulated as a joint RRH selection and power minimization beamforming problem, which takes into account both the backhaul power consumption and RRH transmit power consumption. The backhaul power consumption is determined by the number of active RRHs, while the transmit power consumption of the active RRHs is minimized through coordinated beamforming. This problem is a convex-cardinality optimization problem [10], which is NP-hard. By adopting the branch-and-bound method, we first propose a global optimization algorithm to provide a performance benchmark. We will then propose two low-complexity algorithms yielding the group sparse beamforming design to find the approximately optimal solutions of the joint optimization problem. Through simulations, it is shown that our proposed design framework significantly improves the network energy efficiency, with a performance close to the optimal one.
II. SYSTEM MODEL

Without loss of generality, we consider a homogeneous C-RAN with $L$ RRHs of the same type, each with $N$ antennas, and $K$ single-antenna user terminals. All the RRHs are connected to the BBU pool through backhaul links as shown in Fig. 1. For simplicity, we assume that all the backhaul links are of the same type. Global channel state information (CSI) and all the user data are available at the BBU pool, which enables full cooperation in the network. The BBU pool performs all the baseband signal processing information (CSI) and all the user data are available at the BBU pool, which enables full cooperation in the network.

The BBU pool performs all the baseband signal processing and makes the RRH selection decision. Let $\mathcal{L} = \{1, \ldots, L\}$ and $\mathcal{S} = \{1, \ldots, K\}$ denote the sets of RRH and scheduled user indices, respectively. We denote $\mathcal{A} \subseteq \mathcal{L}$ as the set of active RRHs. Then, the received signal at user $k$ under the complex baseband-equivalent channel model is given by

$$y_k = \sum_{l \in \mathcal{A}} h_{kl}^H w_{lk} s_k + \sum_{i \neq k} \sum_{l \in \mathcal{A}} h_{kl}^H w_{il} s_i + z_k,$$

where $w_{lk} \in \mathbb{C}^{N \times 1}$ is the beamforming vector at RRH $l$ for user $k$ and $h_{kl} \in \mathbb{C}^{N \times 1}$ is the channel vector from RRH $l$ to user $k$, $z_k$ is the white Gaussian noise at user $k$ with distribution $\mathcal{CN}(0, \sigma_k^2)$ and $s_k$ is the data symbol for user $k$ with $E[|s_k|^2] = 1$. We assume that each user applies the single user detection (i.e., treat interference from other users as noise), then the signal-to-interference-plus-noise ratio (SINR) for the user $k$ is given by

$$\text{SINR}_k = \frac{|\sum_{l \in \mathcal{A}} h_{kl}^H w_{lk}|^2}{\sum_{i \neq k} \sum_{l \in \mathcal{A}} |h_{kl}^H w_{il}|^2 + \sigma_k^2}.$$  

Thus, the achievable instantaneous rate at user $k$ is given by

$$r_k = \log(1 + \text{SINR}_k).$$  

We denote $Q_k$ as the demand data rate of user $k$. Thus, we impose the following QoS constraint,

$$r_k \geq Q_k, \quad \forall k \in \mathcal{S}.$$  

The transmit signal at the $l$th RRH can be written as

$$x_l = \sum_{k \in \mathcal{S}} w_{lk} s_k, \quad \forall l \in \mathcal{A}.$$  

Note that the BBU pool will send the baseband signal $x_l$ to the corresponding RRHs through backhaul links. Each RRH has its own power constraint

$$\sum_{k \in \mathcal{S}} |w_{lk}|^2 \leq P_l, \quad \forall l \in \mathcal{A}. \quad (6)$$

A. Network Power Consumption Model

We denote the total RRH transmit power consumption as

$$P_1(A, \mathcal{W}) = \frac{1}{\rho} \sum_{k \in \mathcal{S}} |w_k|^2$$

where $\rho$ denotes the inefficiency coefficient of the power amplifier at each RRH and $\mathcal{W}$ denotes aggregate beamforming vectors $[w_k]_{k \in \mathcal{S}}$ with $w_k = [w_{1k}, \ldots, w_{lk}]^T(A)$. We assume that each backhaul link has a fixed average power consumption $P_c$, then the total backhaul power consumption is given by

$$P_2(A) = |\mathcal{A}| P_c, \quad (8)$$

where $|\cdot|$ denotes the cardinality. To simplify the problem, we ignore the static power consumption at each RRH, such as cooling, digital-to-analog converter, etc., as they can be incorporated into the variable $P_c$. We note that $P_2(A)$ is dynamic and is determined by the number of active RRHs $|\mathcal{A}|$. Therefore, the network power consumption is given by

$$P(A, \mathcal{W}) = P_1(A, \mathcal{W}) + P_2(A). \quad (9)$$

III. PROBLEM FORMULATION AND A GLOBAL OPTIMIZATION SOLUTION

A. Problem Formulation

Since an arbitrary phase rotation of the beamforming vectors $w_k$ does not affect the SINR constraints, objective functions and power constraints, the SINR constraint for user $k$ can be rewritten as a second-order cone (SOC) constraint [11]:

$$Q_k(A, \mathcal{W}) : ||h_k w_1, \ldots, h_k w_K||^T_{\sigma_k} \leq \sqrt{\beta_k} \Re(h_k w_k), \quad (10)$$

where $\Re(\cdot)$ denotes the real part, $\beta_k = 1 + 1/\gamma_k$ with $\gamma_k = 2\beta_k - 1$ and $h_k^T = [h_{1k}, \ldots, h_{lk}](A)$. The per-RRH power constraints (6) can be rewritten as:

$$G_l(A, \mathcal{W}) : |w_l| \leq P_l, \quad l \in \mathcal{A}, \quad (11)$$

where $\bar{w}_l = [w_{l1}, \ldots, w_{lk}]^T \in \mathbb{C}^{K \times 1}$. In the following, when using $G_l(A, \mathcal{W})$, we implicitly indicate that $l \in \mathcal{A}$.

We can now formulate the network power minimization problem with QoS constraints $Q_k(A, \mathcal{W})$ and per-RRH transmit power constraints $G_l(A, \mathcal{W})$ as follows:

$$\mathcal{P} : \text{minimize} \quad P(A, \mathcal{W})$$

subject to \quad $Q_k(A, \mathcal{W}), G_l(A, \mathcal{W}), k \in \mathcal{S}. \quad (12)$

From (9), the backhaul power consumption is determined by $A$, while the RRH transmit power consumption is determined by both $A$ and $\mathcal{W}$. Hence, the problem $\mathcal{P}$ is a joint RRH selection and coordinated power minimization beamforming problem. With a fixed active RRH set $A$, the above optimization is a second-order cone programming (SOCP) problem, which is convex and can be solved in polynomial time. However, the joint problem is a convex-cardinality optimization problem [10], which is NP-hard. Next, we will analyze the problem structure, and then propose a global optimization algorithm.
B. Problem Analysis

The network power consumption consists of two parts: the transmit power consumption (7) and backhaul power consumption (8).

Define the following total backhaul power consumption minimization problem:

$$ P_1 : \text{minimize} \quad P_c |A| $$

subject to \( Q_k(A, W), G_l(A, W), k \in S \). (13)

Define the total transmit power minimization problem with a given active RRH set \( A \) as follows:

$$ P_2(A) : \text{minimize} \quad \frac{1}{\rho} \sum_{k \in S} \| w_k \|^2 $$

subject to \( Q_k(W), G_l(W), k \in S \). (14)

Denote \( P_2^r(A) \) as the optimal value of \( P_2(A) \).

The two optimization problems \( P_1 \) and \( P_2(A) \) are coupled, so the network power optimization problem \( P \) is a joint RRH selection and power minimization beamforming problem. In the next subsection, we will adopt the branch-and-bound method to give the globally optimal solution of \( P \), which, however, has high complexity. In Section IV and V, we will propose two low-complexity algorithms, where the main idea is to decouple the problems \( P_1 \) and \( P_2(A) \).

C. Global Optimization Solution: A Branch-and-Bound Algorithm

In this subsection, we adopt the branch-and-bound method to give the globally optimal solution to the network power minimization problem \( P \). We first reformulate problem \( P \) as a mixed-integer programming problem:

$$ P_3 : \text{minimize} \quad P_c \sum_{l \in \mathcal{L}} z_l + \frac{1}{\rho} \sum_{k \in S} \| w_k \|^2 $$

subject to \( Q_k(\mathcal{L}, W), \| \tilde{w}_l \| \leq z_l \sqrt{P_l}, z_l \in \{0, 1\}, k \in S, l \in \mathcal{L} \). (15)

where \( \mathcal{Z} = \{0, 1\}^L \). We then relax the binary-value constraints to unit interval constraints, i.e., \( 0 \leq z_l \leq 1 \) and obtain the following relaxation problem:

$$ P_4 : \text{minimize} \quad P_c \sum_{l \in \mathcal{L}} z_l + \frac{1}{\rho} \sum_{k \in S} \| w_k \|^2 $$

subject to \( Q_k(\mathcal{L}, W), \| \tilde{w}_l \| \leq z_l \sqrt{P_l}, 0 \leq z_l \leq 1, k \in S, l \in \mathcal{L} \). (16)

This is an SOCP problem. The optimal solution of \( P_4 \) gives a lower bound of the original problem \( P_3 \). For the branch-and-bound procedure, in each iteration, we need to solve a subproblem with a fixed subset of \( \mathcal{Z} \). Then the remaining of \( \mathcal{Z} \) are relaxed to unit intervals. According to the complexity analysis of interior-point methods [12], [13], the complexity of \( P_4 \) is \( O(3.5^{3.5} 5^{3.5}) \). For the worst case, we need \( 2^L \) iterations. Thus, the complexity of the branch-and-bound algorithm is approximated as \( O(2^L 3.5^{3.5} 5^{3.5}) \), which is exponential to \( L \) and cannot be applied in practice. However, this algorithm can provide a performance benchmark for the proposed low-complexity algorithms.

IV. SUCCESSIVE RRH SELECTION ALGORITHM

As we have seen in section III, computing the optimal solution of \( P \) is difficult. In this section, in order to decouple the joint RRH selection and the transmit power minimization beamforming problem, we propose a heuristic algorithm to successively select the RRHs in order, to minimize the total network power consumption. In each iteration, we exploit the sparsity of beamformers at all the active RRHs and set the smallest terms among the concentrated beamformers \( \tilde{w}_l |_{l \in A} \) to be zero.

Denote the iteration number as \( i = 0, 1, 2, \ldots \). At the \( i \)th iteration, \( \mathcal{N}^{(i)} \subset \mathcal{L} \) shall denote the set of inactive RRHs, and \( \mathcal{A}^{(i)} = \mathcal{L} \setminus \mathcal{N}^{(i)} \) denotes the active RRH set. In iteration \( i \), an additional RRH \( r^{(i)} \in \mathcal{A}^{(i)} \) will be added to \( \mathcal{N}^{(i)} \), resulting in a new set \( \mathcal{N}^{(i+1)} \) after this iteration. We set the initial set \( \mathcal{N}^{(0)} = \emptyset \). In our algorithm, once a RRH is added to the set \( \mathcal{N} \), it cannot be removed.

At iteration \( i \), we solve the following power minimization beamforming problem with the given active RRH set \( \mathcal{A}^{(i)} \):

$$ P^{(i)} : \text{minimize} \quad \sum_{l \in \mathcal{A}^{(i)}} \sum_{k \in S} |w_{lk}|^2 $$

subject to \( Q_k(A^{(i)}, W), G_l(A^{(i)}, W), k \in S \). (17)

This is an SOCP problem and the solution is denoted as \( w_{lk}^{(i)}, l \in \mathcal{A}^{(i)}, k \in S \). The network power consumption at the \( i \)th iteration is given by

$$ P^{(i)} = \frac{1}{\rho} \sum_{l \in \mathcal{A}^{(i)}} \sum_{k \in S} |w_{lk}^{(i)}|^2 + P_c |\mathcal{A}^{(i)}|. $$ (18)

Let \( I \) denote the number of iterations, so that \( P^{(I)} \) will be the first time that (17) is infeasible. It is obvious that \( P^{(I+1)}, \ldots, P^{(I)} \) will also be infeasible. The key question is how to select \( r^{(i)} \) after the \( i \)th iteration. We propose the smallest transmission power rule, i.e., \( r^{(i)} \) is given by

$$ r^{(i)} = \arg \min_{l \in \mathcal{A}^{(i)}} \sum_{k \in S} |w_{lk}^{(i)}|^2. $$ (19)

This rule actually exploits the group sparsity of the aggregative beamformers \( \tilde{w}_l = [w_{l1}^{T}, \ldots, w_{lK}^{T}]^T, l \in \mathcal{A}^{(i)} \) at the \( i \)th iteration. In other words, we set the smallest terms among the aggregative beamformers \( \tilde{w}_l |_{l \in \mathcal{A}^{(i)}} \) to be zero, which gives an inactive RRH \( r^{(i)} \). The algorithm is summarized in the following as Algorithm 1. Note that \( I = 0 \) indicates that the

### Algorithm 1: Successive RRH Selection Algorithm

**Step 1:** Initialize \( \mathcal{N}^{(0)} = \emptyset, \mathcal{A}^{(0)} = \{1, \ldots, L\} \) and \( i = 0 \).

**Step 2:** Solve the optimization problem \( P^{(i)} \), obtain the optimal solutions \( w_{lk}^{(i)} \) and \( P^{(i)} \).

   1) If it is feasible, find the \( r^{(i)} \) (19) and update \( \mathcal{N}^{(i+1)} = \mathcal{N}^{(i)} \cup \{r^{(i)}\} \) and \( i = i + 1 \), and then go to **Step 2**.

   2) If it is infeasible, denote \( I = i \) and go to **Step 3**.

**Step 3:** Obtain the minimum network power consumption \( P^* = \min \{P^{(0)}, P^{(1)}, \ldots, P^{(I-1)}\} \).

network can not support the QoS requirements even with all the RRHs active.
A. Complexity Analysis and Discussion

At each iteration, we need to solve the convex optimization problem $P^{(i)}$ with the complexity of $O((L-i)^{3.5}N^{3.5}K^{3.5})$. For the worst case, we need to solve $L-1$ SOCP convex optimization problems $P^{(i)}$. Therefore, the complexity of the proposed successive RRH selection algorithm is approximated as $O(L^{4.5}N^{3.5}K^{3.5})$. In order to further enhance the efficiency of the algorithm, it is interesting to exploit the intimate relationship between the problems $P^{(i)}$ and $P^{(i+1)}$, since they only differ in the additional element $r^{(i)}$.

V. GROUP-SPARSITY INDUCING NORM BASED ALGORITHMS

In this section, we propose a two-stage strategy to decouple the joint optimization problem, inspired by the fact that the backhaul power consumption is comparable to the RRH transmit power consumption. To do so, we begin by minimizing the number of active RRHs by explicitly exploiting the group-sparsity of the aggregate beamforming vectors $\tilde{w} = [\tilde{w}_l]_{l \in \mathcal{L}}$. After obtaining the active RRH set $\mathcal{A}$, we will further minimize the total transmit power $P_2(\mathcal{A})$.

A. Minimize the Cardinality of the Active RRH Set $\mathcal{A}$

Define $t_l = ||\tilde{w}_l||$ and $t = [t_1, \ldots, t_L]$. Then the backhaul power consumption minimization problem $P_3$ can be reformulated as

$$P_6 : \begin{align*}
\text{minimize} & \quad P_c ||t||_0 \\
\text{subject to} & \quad Q_k(\mathcal{L}, \mathcal{W}), G_l(\mathcal{L}, \mathcal{W}), k \in \mathcal{S}. \quad (20)
\end{align*}$$

where $||t||_0 = ||\{ l : t_l \neq 0 \}||$ counts the number of non-zero coefficients of $t$. If we fix the sparsity pattern of $t$ (i.e., which elements are zero/non-zero), we get an SOCP convex problem. However, the joint problem is actually a convex-cardinality problem [10], which is NP-hard. In the compressive sensing literatures [14], [15], $\ell_1$-norm is well known as a good convex approximation of $\ell_0$-norm. However, the additional constraints $Q_k(\mathcal{L}, \mathcal{W})$ and $G_l(\mathcal{L}, \mathcal{W})$ prevent direct applying the algorithms in [14], [15], which are only for unconstrained problems. Moreover, related to the RRH selection, we are more interested in the group sparsity, i.e., the whole beamforming vector of an RRH being 0 rather than some individual coefficients.

In order to induce the sparsity of $\tilde{w}_k = [\tilde{w}_l]_{l \in \mathcal{L}}$, which results in the RRH selection, we introduce the following group sparsity-inducing norms [16], i.e., the mixed $\ell_{1,q}$ norm:

$$\Omega(\tilde{w}) = \sum_{l \in \mathcal{L}} \left( \sum_{k \in \mathcal{S}} \sum_{n \in \mathcal{N}} |w_{lk}(n)|^q \right)^{1/q}, \quad q > 1, \quad (22)$$

to approximate $||t||_0$, where $\mathcal{N} = \{1, \ldots, N \}$ and $w_{lk}(n)$ is the $n$th element of the beamforming vector $w_{lk}$. The details and geometric interpretations of the group-sparsity norms can be found in [16]. In this paper, we investigate two most popular group-sparsity inducing norms, i.e., mixed $\ell_{1,2}$-norm and $\ell_{1,\infty}$-norm to yield a group sparse beamforming design.

1) Mixed $\ell_{1,2}$-Norm Relaxation: Let $q = 2$, we get the following group sparsity-inducing norm, i.e., the mixed $\ell_{1,2}$-norm

$$\Omega(\tilde{w}) = \sum_{l \in \mathcal{L}} ||\tilde{w}_l||_2 \quad (23)$$

It behaves like an $\ell_1$-norm on the vector $[||\tilde{w}_l||_2]_{l \in \mathcal{L}}$. As a result, $\Omega$ induces the group sparsity. That is to say, each $||\tilde{w}_l||_2$, and equivalently each $\tilde{w}_l$, is encouraged to be set to zero. In other words, each RRH $l$ is encouraged to be turned off. Therefore, by adopting the mixed $\ell_{1,2}$-norm of (23) as a convex surrogate of the $\ell_0$-norm in $P_5$, the backhaul power consumption minimization problem can be relaxed as

$$P_6 : \begin{align*}
\text{minimize} & \quad P_c \sum_{l \in \mathcal{L}} ||\tilde{w}_l||_2 \\
\text{subject to} & \quad Q_k(\mathcal{L}, \mathcal{W}), G_l(\mathcal{L}, \mathcal{W}), k \in \mathcal{S}. \quad (24)
\end{align*}$$

We shall now introduce auxiliary variables $\alpha_l, l = 1, \ldots, L$. The above optimization problem can then be reformulated as an SOCP problem:

$$P_7 : \begin{align*}
\text{minimize} & \quad P_c \sum_{l \in \mathcal{L}} \alpha_l \\
\text{subject to} & \quad Q_k(\mathcal{L}, \mathcal{W}), G_l(\mathcal{L}, \mathcal{W}), \quad ||\tilde{w}_l||_2 \leq \alpha_l, l \in \mathcal{L}, k \in \mathcal{S}. \quad (25)
\end{align*}$$

2) Mixed $\ell_{1,\infty}$-Norm Relaxation: In the last subsection, we introduced the mixed $\ell_{1,2}$-norm to induce the group sparsity. However, within the groups $l \in \mathcal{L}$, the $\ell_2$-norm of $\tilde{w}_l$ does not promote sparsity within the vector $\tilde{w}_l = [w_{l1}(1), \ldots, w_{lk}(N)]^T$. Furthermore, the tightest convex positively homogeneous lower bound of the objective function of the original problem $P$ can be given as [17]

$$\Theta(\tilde{w}) = \frac{1}{2} \sqrt{P_c \rho \sum_{l \in \mathcal{L}} ||\tilde{w}_l||}_2 \quad (26)$$

Therefore, the solution of $P_6$ gives an approximate solution of the original problem $P$ scaled by a constant, but it may not be the sparsest solution of the beamformers. As our goal in this step is to minimize the number of active RRHs, which needs the sparsest solution of the beamformers, we introduce the following mixed $\ell_{1,\infty}$-norm, to further induce the sparsity in each group $l$:

$$\Omega(\tilde{w}) = \sum_{l \in \mathcal{L}} \max_{k \in \mathcal{S}, n \in \mathcal{N}} |w_{lk}(n)| \quad (27)$$

As a result, each element $w_{lk}(n)$ at the same group $\tilde{w}_l$ is encouraged to be set to zero. Thus, using the mixed $\ell_{1,\infty}$-norm as a convex approximation to the $\ell_0$-norm in $P_5$, the backhaul power consumption minimization problem can be relaxed as

$$P_7 : \begin{align*}
\text{minimize} & \quad P_c \sum_{l \in \mathcal{L}} \max_{k \in \mathcal{S}, n \in \mathcal{N}} |w_{lk}(n)| \\
\text{subject to} & \quad Q_k(\mathcal{L}, \mathcal{W}), G_l(\mathcal{L}, \mathcal{W}), k \in \mathcal{S}. \quad (28)
\end{align*}$$

Next we introduce the auxiliary variables $\alpha_{l,k,n}, l \in \mathcal{L}, k \in \mathcal{S}, n \in \mathcal{N}$. The above optimization problem can then be reformulated as:

$$P_7 : \begin{align*}
\text{minimize} & \quad P_c \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{S}, n \in \mathcal{N}} \alpha_{l,k,n} \max_{k \in \mathcal{S}, n \in \mathcal{N}} |w_{lk}(n)| \\
\text{subject to} & \quad Q_k(\mathcal{L}, \mathcal{W}), G_l(\mathcal{L}, \mathcal{W}), \quad ||\tilde{w}_l(l,n)|| \leq \alpha_{l,k,n}, l \in \mathcal{L}, k \in \mathcal{S}. \quad (29)\end{align*}$$
where $w_k(l, n) = w_{l_k}(n)$.

3) **RRH Selection:** After solving the relaxation problems $P_6$ or $P_7$, we would obtain the transmit power $t^*_l$ of each RRH. Let $\pi$ denote the permutation that sorts $t^*_l$ in the ascending order, i.e., $t^*_\pi_1 \leq t^*_\pi_2 \cdots \leq t^*_\pi_L$. We adopt the bisection search method [12] to efficiently find $M$, i.e., the minimum number of active RRHs that makes the following problem feasible:

$$
F^{(i)} : \quad \text{find } W
$$
subject to $Q_k(A^{(i)}, W), G_l(A^{(i)}, W), k \in S$. (30)

where $A^{(i)} = L \setminus \{\pi_1, \ldots, \pi_i\}$ with $\pi_0 = \emptyset$.

### B. Total Transmit Power Minimization

In the last subsection, we applied the group-sparsity inducing norms to approximately find the minimum number of active RRHs. After obtaining the approximately optimal solution $A^*$ of $P_5$ with $|A^*| = M$, we can minimize the total transmit power $P_2(A^*)$ as a second stage and obtain the minimum transmit power $P_2^*(A^*)$. Finally, we get the minimum network power consumption

$$
P^* = MP_c + P_2^*(A^*).$$

The proposed group-sparsity inducing norm based algorithms are summarized as follows:

#### Algorithm 2: Group-Sparsity Inducing Norm Based Algorithms

**Step 1:** Solve the optimization problem $P_6$ or $P_7$, obtain the RRHs transmit power $t^*_l$ and sort them in the ascending order: $t^*_\pi_1 \leq t^*_\pi_2 \cdots \leq t^*_\pi_L$.

**Step 2:** Initialize $M_{\text{low}} = 0, M_{\text{up}} = L$.

**Step 3:** Repeat

1) Set $i \leftarrow \lfloor \frac{M_{\text{low}} + M_{\text{up}}}{2}\rfloor$.
2) Check the feasibility problem $F^{(i)}$: if it is feasible, set $M_{\text{low}} \leftarrow i$; otherwise, set $M_{\text{up}} \leftarrow i$.

**Step 4:** Until $M_{\text{up}} - M_{\text{low}} = 1$, obtain $M = M_{\text{low}}$ and the optimal active RRH set $A^* = L \setminus \{\pi_1, \ldots, \pi_M\}$.

**Step 5:** Solve the total transmit power minimization problem $P_2^*(A^*)$ and obtain the minimum network power consumption $P^*$ (31).

### C. Complexity Analysis and Discussions

In this algorithm, we only need to solve the SOCP problem $P_6$ or $P_7$. Therefore, the complexity of the group-sparsity norms based algorithm is approximated as $O\left(L^{3.5}N^{3.5}K^{3.5}\right)$. It is noted that there is another motivation for minimizing the number of active RRHs, which is related to the CSI acquisition. If we can determine the active RRH set only based on the statistic CSI, it can significantly reduce the CSI feedback overhead. Investigating the problem $P_6$ or $P_7$ only based on the statistic CSI is an interesting research direction.

### VI. Simulation Results

In this section, we will simulate the performance of the proposed algorithms. We model the path and penetration loss as $127 + 25\log_{10}(d)$ with $d$ (km) as the propagation distance, the small scale fading is modeled as independent circularly symmetric complex Gaussian random variables with distribution $CN(0, 1)$, and the noise power spectral density is -174 dBm/Hz. The maximum RRH transmit power is $P_t = 1W, l \in L$, the backhaul link power consumption for each fiber is $P_c = 5W$ and the inefficiency coefficient is $\rho = 0.2$.

We first consider a network with $L = 4$ RRHs, each equipped with $N = 2$ antennas, and $K = 5$ mobile users. The deployment of the BSs and users are shown in Fig. 2, and their positions are fixed during all the simulations. In Fig.

![Fig. 2. Remote radio heads and mobile users deployment with fixed RRH and MU positions.](image-url)
Fig. 3. Average network power consumption using different algorithms under a fixed network topology, as shown in Fig. 2.

Fig. 4. Remote radio heads and mobile users deployment with fixed RRH positions and random MUs positions.

RRH selection. Both Fig. 3 and Fig. 5 indicate that the successive RRH selection algorithm is better than the \( \ell_{1,2} \)-norm based algorithm. The reason is that the optimization problem \( P^{(0)} \) in (17) is equivalent to the transmit power optimization problem \( P_0 \) in (24). The \( \ell_{1,2} \)-norm based algorithm selects the RRHs only based the solution of \( P_0 \). On the other hand, the successive RRH selection algorithm selects only one RRH at each iteration. Thus, the simulation results show that iteratively selecting the RRHs is an appealing strategy to minimize the network power consumption. Both Fig. 3 and Fig. 5 also show that Algorithm 2 with \( \ell_{1,\infty} \)-norm relaxation is better than the \( \ell_{1,2} \)-norm relaxation. The reason is that the backhaul power consumption is dominant in C-RAN and \( \ell_{1,\infty} \)-norm based algorithm yields the sparsest solution of the beamformer, (as explained in Section V. A), resulting in fewer active RRHs.

VII. CONCLUSIONS

In this paper, we proposed a new framework to improve the energy efficiency of cellular networks with the new architecture of C-RAN. It was shown that the backhaul power consumption cannot be ignored when designing green C-RAN. Besides providing a global optimization algorithm to minimize the network power consumption, we also connected the design problem to the group-sparse beamforming problem, based on which, two low-complexity algorithms were proposed. Simulation results showed that the proposed group sparse beamforming framework provides an effective way to reduce the network power consumption.

REFERENCES