Centre Tecnològic de Telecomunicacions de Catalunya

UNIVERSITAT POLITÈCNICA DE CATALUNYA

Impact of channel state information on the analysis and design of multi-antenna communication systems

(PhD dissertation)

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General overview

- **Objectives:**
  - Analysis and design of multi-antenna systems
    - Analysis in terms of achievable rates and capacity (ultimate performance limits)
    - Design carried out according to practical criteria
      - Mean squared error
      - Received symbols distance
  - Characterize the impact of the CSI (Available information on the channel state)

Outline

- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)
- Conclusions (Ch. 7)

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- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)

Most simple case: single user communications with perfect CSI

–**Contribution: DESIGN** of a linear transmitter to maximize the minimum distance among the received constellation points

- Conclusions (Ch. 7)

Outline

- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)
  - **Perfect CSI and single user communications**
- Conclusions (Ch. 7)

Outline

- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)

Assuming perfect CSI is rather unrealistic (specially in practical deployments)

We studied communication systems with power feedback

This evolved into capacity analysis with magnitude knowledge and phase uncertainty

–**Contribution**: **ANALYSIS** of the capacity for this kind of CSI

- Conclusions (Ch. 7)

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  - **Incomplete CSI and single user communications**
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- Communications set-up (Ch. 2)
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One of the main advantages of multi-antenna systems is their multiuser capabilities

We decided to analyze the achievable rates of the THP because it unifies the single and multiuser formulations

–**Contribution**: **ANALYSIS** of the achievable rates for THP in the presence of errors in the CSI

- Conclusions (Ch. 7)

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  - Incomplete CSI and single user communications
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The results were disappointing, because they did not represent a great enhancement with respect to the existing literature. It seemed that THP was a limiting architecture

Fully focus on the design of a linear transmitter for a multiuser system with imperfect CSI.

–**Contribution**: **DESIGN** of a robust linear transmitter to guarantee QoS requirement and transmit the minimum power

- Conclusions (Ch. 7)

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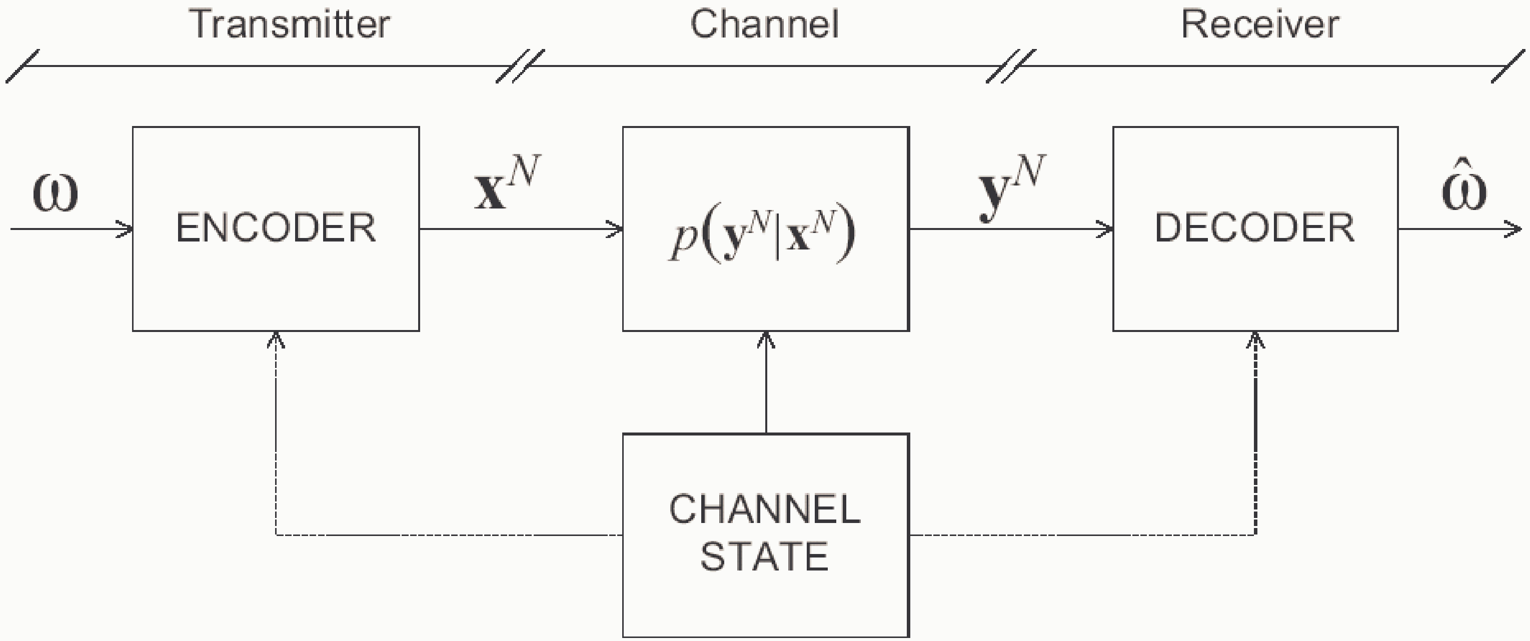
Communications set-up

- Single user scenario

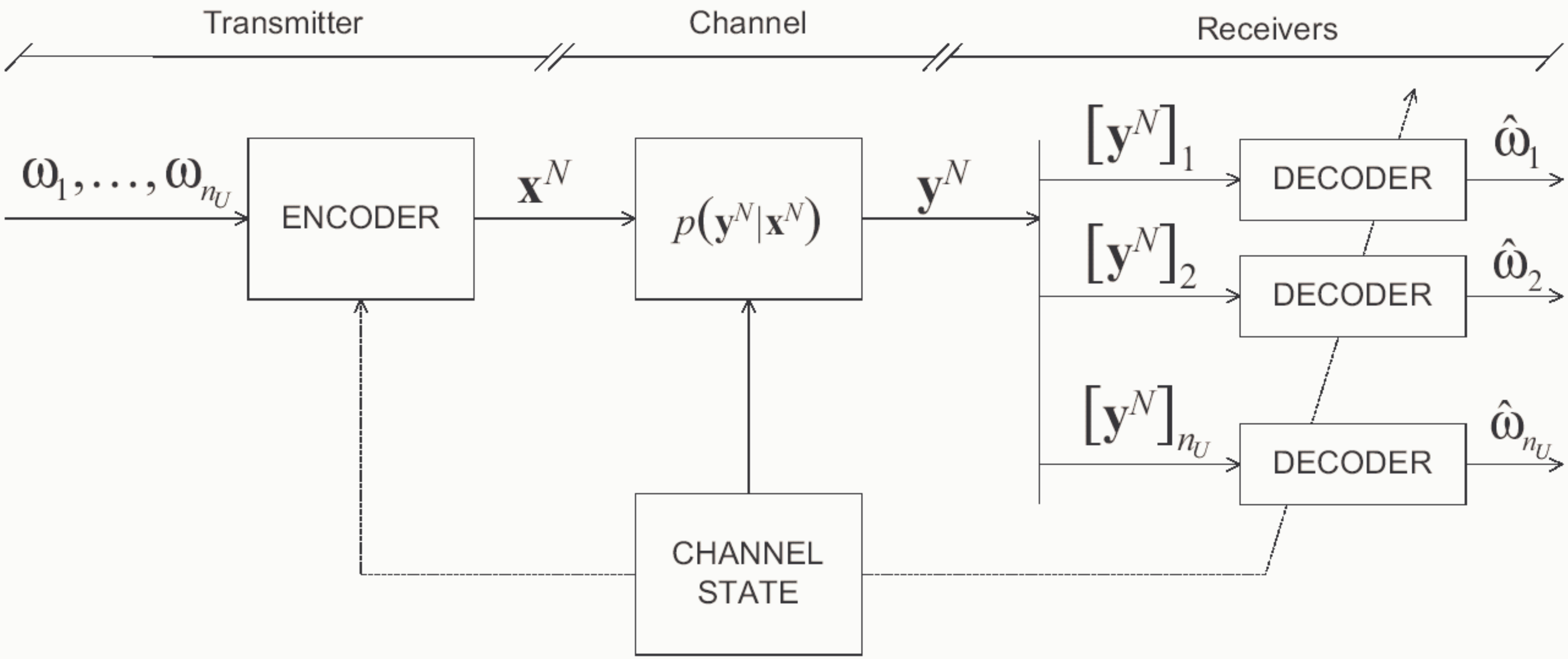


$\left[\mathbf{x}^N\right]_j$ Signal transmitted through each one of the antennas

$\left[\mathbf{y}^N\right]_i$ Signal received through each one of the antennas

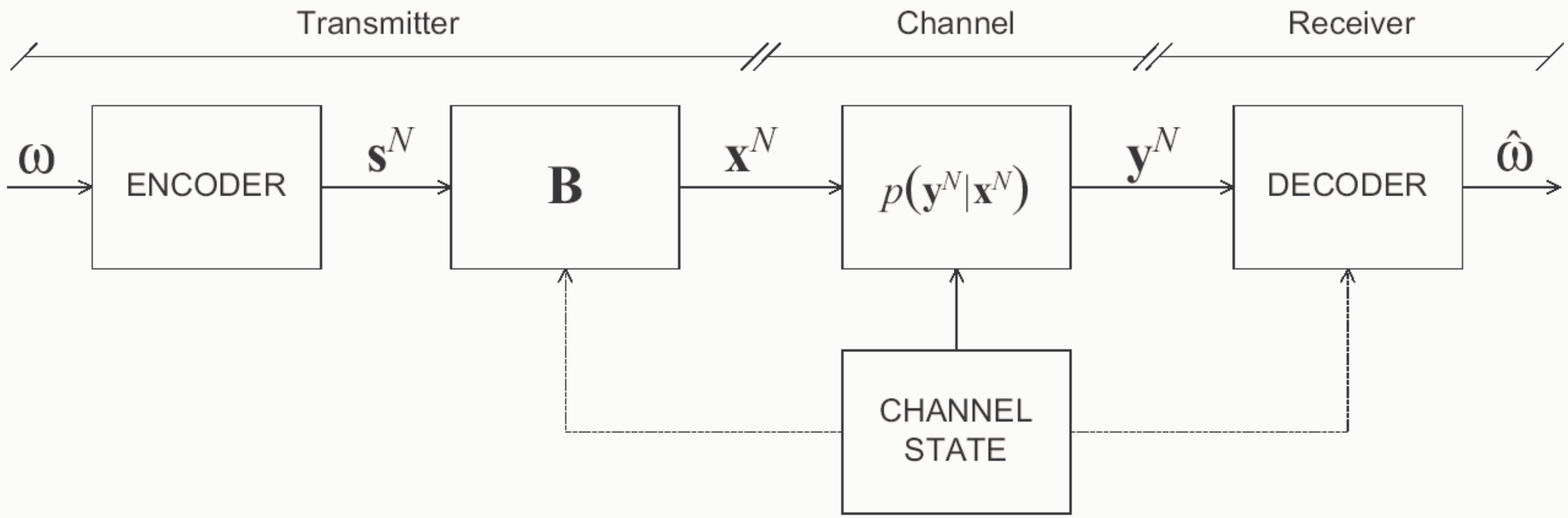
Communications set-up

- Multiuser scenario (broadcast)



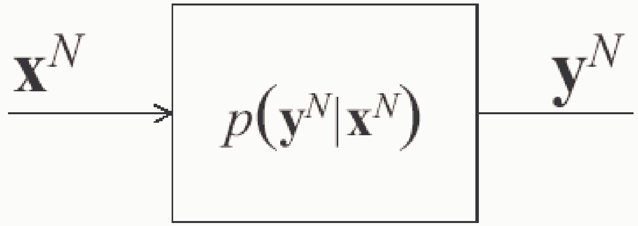
Communications set-up

- Linear transmitter scenario

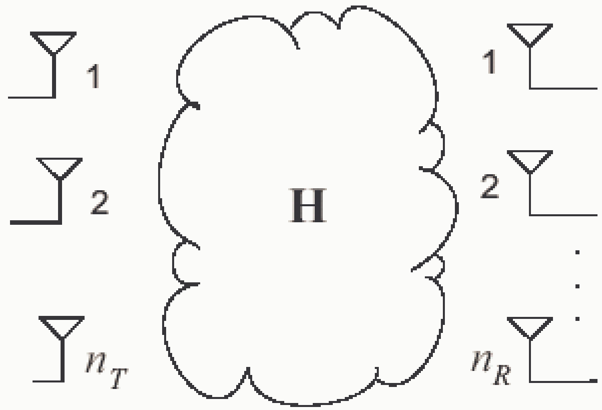


Communications set-up

- The multiple-antenna channel (MIMO)



  - Memoryless $p\left(\mathbf{y}^N|\mathbf{x}^N\right) = \prod_{i=1}^{N} p(\mathbf{y}_i|\mathbf{x}_i)$



  - Linear $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$
  - Corrupted with additive white Gaussian noise

$$\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_n) \qquad \mathbb{E}\mathbf{n} = \mathbf{0} \qquad \mathbb{E}\mathbf{n}\mathbf{n}^H = \mathbf{R}_n$$

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Cases of study

- Different degrees of CSI

Perfect CSI

Optimal linear filtering

Imperfect CSI

Robust designs

Incomplete CSI

Statistical design

Maximin design

No CSI

Space-time codes

Cases of study

- Different degrees of CSI

Perfect CSI

Optimal linear filtering

Imperfect CSI

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Incomplete CSI

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1 - Capacity results (by Telatar)

- The transmitter can be matched to each channel realization

$$C = \max_{\mathbf{Q}} \quad \log\det\left(\mathbf{I} + \mathbf{R}_n^{-1}\mathbf{H}\mathbf{Q}\mathbf{H}^H\right) \qquad \mathbf{Q} = \mathbb{E}\mathbf{x}\mathbf{x}^H$$

- The optimal **Q** is obtained from the SVD decomposition

$$\mathbf{Q}^\star = \mathbf{U}_\mathbf{H}\mathbf{\Lambda}_\mathbf{Q}\mathbf{U}_\mathbf{H}^H$$

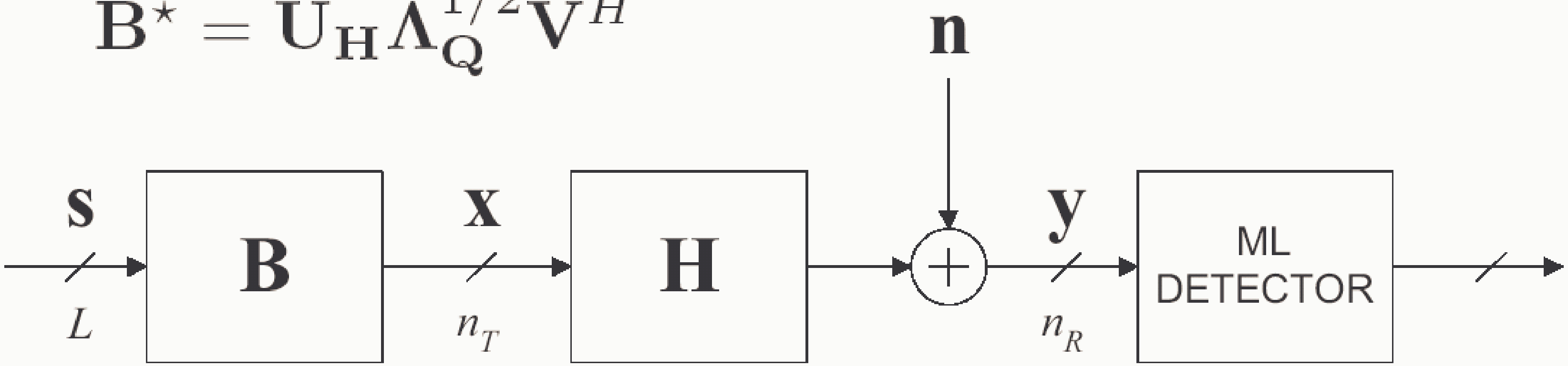
where

$$\lambda^\star_{\mathbf{Q},j} = \left(\mu - \frac{1}{\lambda_{\mathbf{H},j}}\right)^+$$

1 - Capacity results

- Since the optimal covariance matrix is $\mathbf{Q}^{\star} = \mathbf{U_H \Lambda_Q U_H}^H$ a capacity achieving structure is

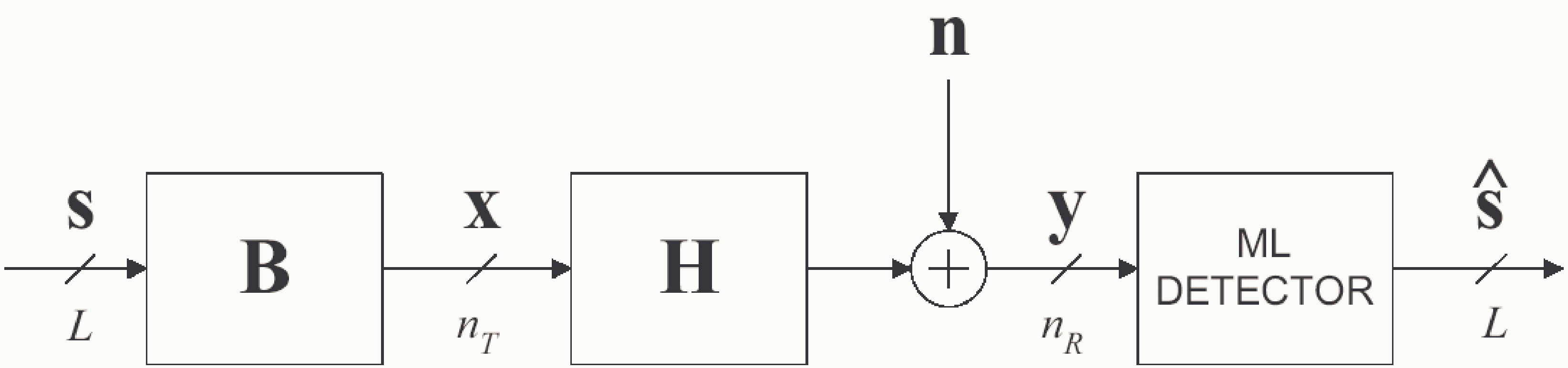
$$\mathbf{B}^{\star} = \mathbf{U_H}\mathbf{\Lambda}_{\mathbf{Q}}^{1/2}\mathbf{V}^H$$



- The capacity achieving codes are practically unfeasible
  - Infinite length
  - Gaussian distributed

1 - Practical transmitter design

- We want to solve the problem for the case where
  - We are transmitting a given constellation (BPSK, QPSK,...)
  - The detection is performed in a symbol by symbol basis



- The detection rule is

$$\hat{\mathbf{s}} = \arg\max_{\mathbf{s}\in\mathcal{C}} \log \mathcal{L}(\mathbf{y}, \mathbf{B}\mathbf{s})$$

1 - Problem statement

- The transmitted constellation, **Bs**, and the received one, **HBs**, are related by the linear transformation of the channel

- With ML detection, the performance is dominated by the worst pair-wise error probability:

$$\mathrm{pep}_{n,m} = \Pr\left(\mathcal{L}(\mathbf{s}_m) > \mathcal{L}(\mathbf{s}_n) | \mathbf{s}_n\right) = \mathcal{Q}\left(\sqrt{\frac{d^2_{n,m}}{2\sigma^2}}\right)$$

  where $d^2_{n,m}$ is the distance between two **received** constellation points, $d^2_{n,m} = \|\mathbf{HBs}_n - \mathbf{HBs}_m\|^2$

1 - Problem statement

- Our optimization criterion is

$$\arg\min_{\mathbf{B}} \max_{q} \ \mathsf{pep}_q = \arg\max_{\mathbf{B}} \min_{q} \ d_q^2 = \arg\max_{\mathbf{B}} \ d_{\min}^2$$

where the squared distance is given by

$$d_{n,m}^2 = \|\mathbf{H}\mathbf{B}\mathbf{s}_n - \mathbf{H}\mathbf{B}\mathbf{s}_m\|^2 = \mathbf{e}_{n,m}^H \mathbf{B}^H \mathbf{R}_{\mathbf{H}} \mathbf{B} \mathbf{e}_{n,m}$$

with $\mathbf{R}_{\mathbf{H}} = \mathbf{H}^H \mathbf{H}$ and $\mathbf{e}_{n,m} = \mathbf{s}_n - \mathbf{s}_m$

- We obtain

$$\begin{aligned} \max_{\mathbf{B}} \ \min_{\mathbf{e}} \quad & \mathbf{e}^H \mathbf{B}^H \mathbf{R}_{\mathbf{H}} \mathbf{B} \mathbf{e}, \\ \text{s.t.} \quad & \mathbf{e} \in \mathcal{E}, \\ & \mathrm{Tr}\, \mathbf{B}\mathbf{B}^H \leq P_T \end{aligned}$$

1 - Problem solution

- The optimal transmitter matrix is $\mathbf{B}^{\star} = \mathbf{U}_{\mathbf{H}} \mathbf{\Lambda}_{\mathbf{Q}}^{1/2} \mathbf{V}^{H}$

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- The optimal transmitter matrix is $\mathbf{B}^\star = \boxed{\mathbf{U}_{\mathbf{H}}}\boldsymbol{\Lambda}_{\mathbf{Q}}^{1/2}\mathbf{V}^H$
- The left eigenvectors are the same as in the capacity achieving solution
  - Transmission along the eigenmodes of the channel

1 - Problem solution

- The optimal transmitter matrix is $\mathbf{B}^\star = \mathbf{U_H} \boxed{\mathbf{\Lambda_Q}^{1/2}} \mathbf{V}^H$
- The power allocation is completely different

$$\lambda^\star_{\mathbf{Q},j} = \frac{t^\star \beta_j}{\lambda_{\mathbf{H},j}} \qquad\qquad \lambda^\star_{\mathbf{Q},j} = \left(\mu - \frac{1}{\lambda_{\mathbf{H},j}}\right)^+$$

  - Similar to the ZF solution (inversion of the gains)
  - The parameter t is calculated to satisfy the power constraint
  - The main difference is the presence of the beta factors
    - Can be equal to zero → Some eigenmodes are disabled
    - For a given active set, they are constant values, which depend on the geometry of the constellation
    - The beta factors define the aspect ratio of the received constellations

1 - Problem solution

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$$\lambda_{\mathbf{Q},j}^{\star} = \frac{t^{\star}\beta_j}{\lambda_{\mathbf{H},j}} \qquad \lambda_{\mathbf{Q},j}^{\star}\lambda_{\mathbf{H},j} = t^{\star}\beta_j \qquad \lambda_{\mathbf{Q},j}^{\star} = \left(\mu - \frac{1}{\lambda_{\mathbf{H},j}}\right)^{+}$$

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  - The parameter t is calculated to satisfy the power constraint
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1 - Problem solution

- The optimal transmitter matrix is $\mathbf{B}^{\star} = \mathbf{U_H}\mathbf{\Lambda}_{\mathbf{Q}}^{1/2}\boxed{\mathbf{V}^{H}}$

- The right eigenvectors matrix $\mathbf{V}$ is very difficult to calculate (in general, exhaustive search is needed). The purpose of this matrix is to linearly tranform the constellation points in $\mathbf{s}$ into a new constellation $\mathbf{Vs}$

1 - Problem solution for a simple case

- Two QPSK data streams are to be sent (2 eigenmodes)
- The optimal solution has a closed form expression

- One active stream

$$\lambda_{\mathbf{H},2}/\lambda_{\mathbf{H},1} < 0.097$$

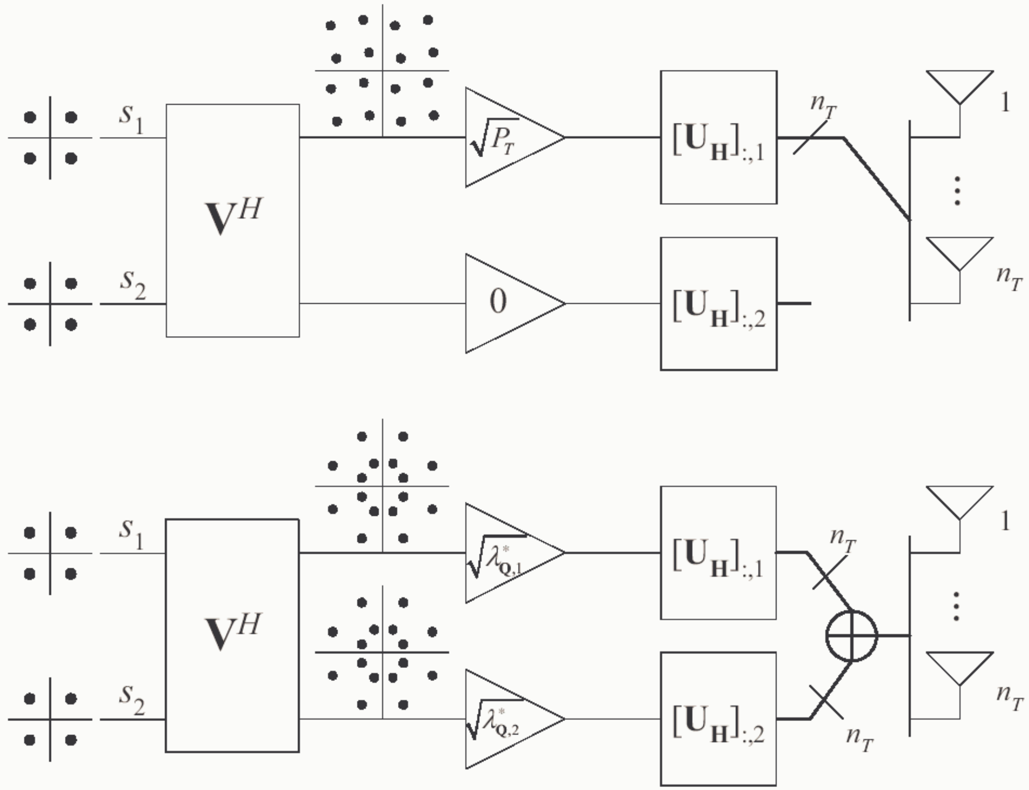
$$\beta_1 = 1 \qquad \beta_2 = 0$$

- Two active streams

$$\lambda_{\mathbf{H},2}/\lambda_{\mathbf{H},1} > 0.097$$

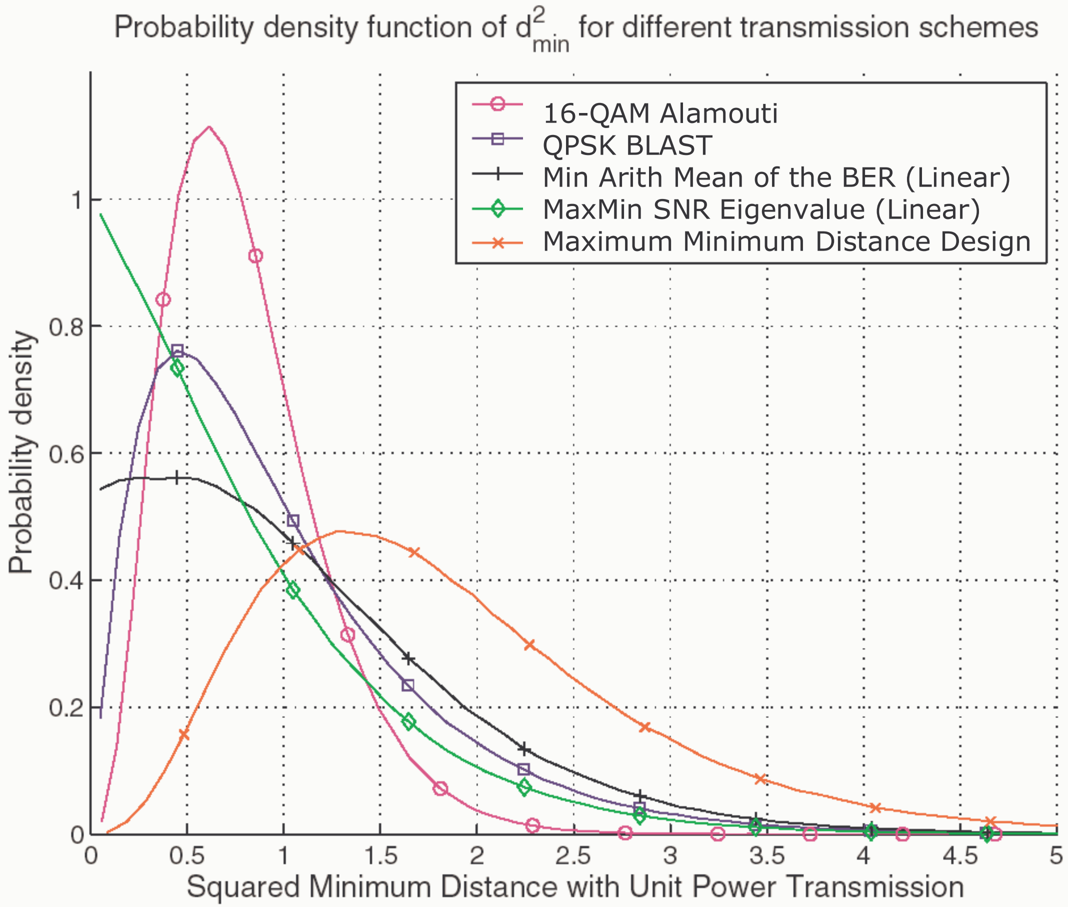
$$\beta_1 = 1$$

$$\beta_2 = 3 - 2\sqrt{2}$$



1 - Performance evaluation

- Comparison for 4 bit per channel use (2QPSK streams)



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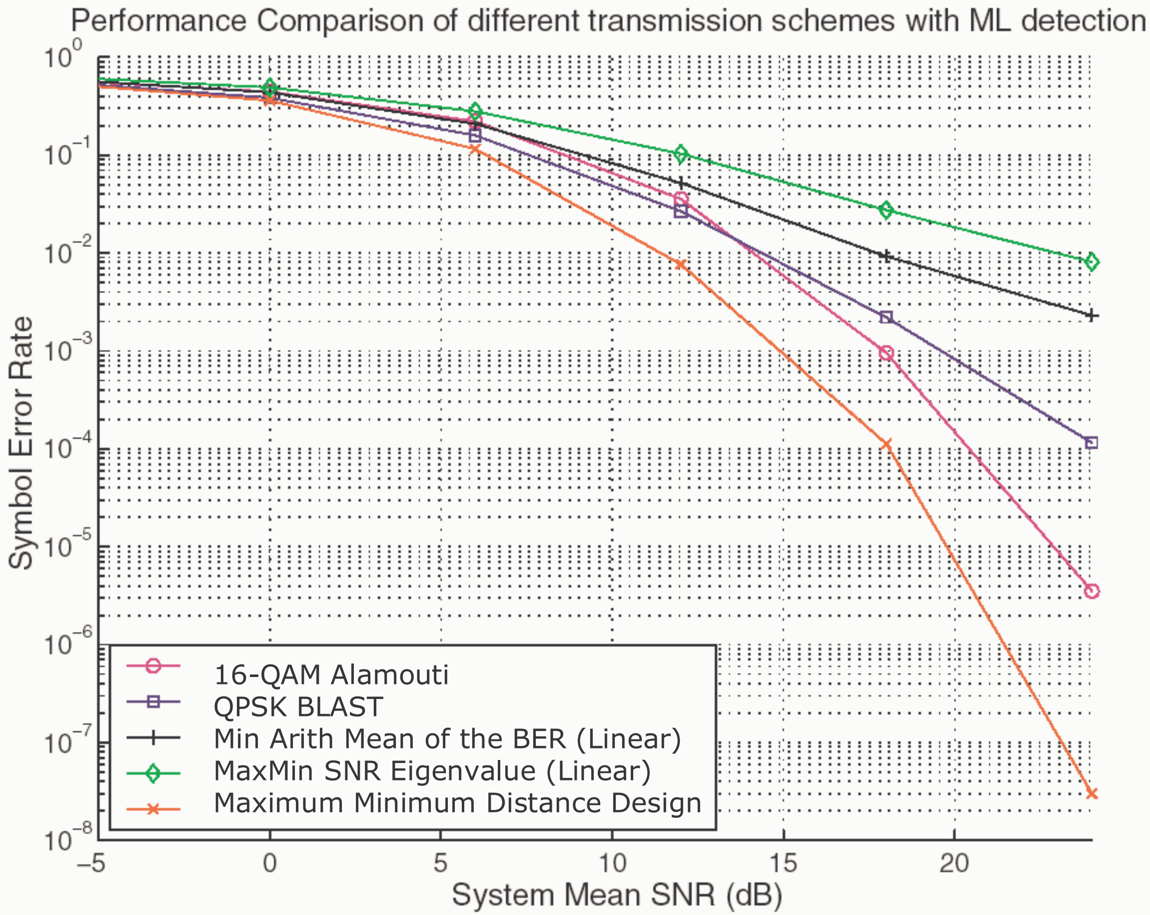

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2 - Capacity results

- Single user MIMO capacity with incomplete CSI has been studied for the cases of statistical knowledge where the CSI consisted of:
  - Channel covariance and/or mean knowledge

- Motivated by system design with power feedback, we consider a different kind of incomplete CSI, which in this case is not statistical but instantaneous:
  - Magnitude knowledge
  - Phase uncertainty

$$[\mathbf{H}]_{ij} = m_{ij}\, \mathrm{e}^{\mathrm{i}\,\theta_{ij}}$$

$$\mathbf{H} = \mathbf{M} \odot \mathbf{P}$$

2 - Uncertainty model application

- Dynamic scenarios (Ergodic formulation)
  - Scenario with direct line of sight, where the mobile user is moving slowly and there exists moderate to high uncorrelation of the channel phases as described in [Paulraj,Kailath,88]
  - While the magnitude matrix **M** would remain approximately constant, the phases in **P** would change very rapidly because of the relative movement

- Static scenarios (Compound formulation)
  - Scenario where no significant channel variability may occur during the transmission of the message and where the transmitter is only able to estimate accurately the magnitude matrix **M**. For example, due to different track lengths in the RF front ends at each antenna

2 - Capacity results

- Instantaneous mutual information

$$\Psi(\mathbf{Q},\mathbf{M},\mathbf{P}) = \log\det\left(\mathbf{I} + \sigma^{-2}(\mathbf{M}\odot\mathbf{P})\mathbf{Q}(\mathbf{M}\odot\mathbf{P})^H\right)$$

- Ergodic formulation
  - Mutual information $I_E(\mathbf{Q},\mathbf{M}) = \mathbb{E}_{\mathbf{P}}\Psi(\mathbf{Q},\mathbf{M},\mathbf{P})$
  - Capacity $C_E = \sup_{\mathbf{Q}} \quad I_E(\mathbf{Q},\mathbf{M})$
- Compound formulation
  - Mutual information $I_C(\mathbf{Q},\mathbf{M}) = \inf_{\mathbf{P}\in\mathcal{P}} \Psi(\mathbf{Q},\mathbf{M},\mathbf{P})$
  - Capacity $C_C = \sup_{\mathbf{Q}} \quad I_C(\mathbf{Q},\mathbf{M})$

2 - Capacity results

- For both cases (ergodic and compound) the optimal covariance matrix is diagonal (power allocation)

$$\mathbf{Q}^{\star} = \mathbf{\Lambda}_{\mathbf{Q}}$$

- Since no phase information is available, the eigenmodes of the channel are not well defined
- It seems reasonable not to choose any privileged direction

- Formally, the optimality of the diagonal structure is proven using the invariance properties of the mutual information functions

2 - Ergodic formulation

- Closed form solution exists for the 2x2 case
- For the general case the solution has to be calculated numerically

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \mathbb{E}_\mathbf{P} \Psi(\mathbf{\Lambda_Q}, \mathbf{M}, \mathbf{P}) \qquad C_E = \sup_{\mathbf{\Lambda_Q}} \quad I_E(\mathbf{\Lambda_Q}, \mathbf{M})$$

- The objective function is difficult to be evaluated
  - Finite sample solution (Set of phases $\mathcal{M} \subset \mathcal{P}$)

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) \simeq I_E^{\mathrm{num}}(\mathbf{\Lambda_Q}, \mathbf{M}) = \frac{1}{|\mathcal{M}|} \sum_{\mathbf{P} \in \mathcal{M}} \Psi(\mathbf{\Lambda_Q}, \mathbf{M}, \mathbf{P})$$

  - Results from RMT

$$\lim_{\substack{n_T \to \infty \\ n_R \to \infty}} I_E(\mathbf{\Lambda_Q}, \mathbf{M}) - \bar{I}_E(\mathbf{\Lambda_Q}, \mathbf{M}) = 0, \text{ a.s.}$$

2 - Ergodic approximation with RMT

- Basic development (follows the procedure of W. Hachem)

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \mathbb{E}_\mathbf{P} \log\det\left(\mathbf{I} + \sigma^{-2}(\mathbf{M} \odot \mathbf{P})\mathbf{\Lambda_Q}(\mathbf{M} \odot \mathbf{P})^H\right)$$

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \mathbb{E}_\mathbf{P} \sum_{i=1}^{n_R} \log\left(1 + \sigma^{-2}\zeta_i\right)$$

Stieltjes transform of the random probability measure of the eigenvalues

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \int_{\sigma^2}^{\infty} \left(\frac{1}{\xi} - \mathbb{E}_\mathbf{P} m_\mu(-\xi)\right) \mathrm{d}\xi$$

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) \simeq \bar{I}_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \int_{\sigma^2}^{\infty} \left(\frac{1}{\xi} - m_\nu(-\xi)\right) \mathrm{d}\xi$$

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$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \mathbb{E}_{\mathbf{P}} \sum_{i=1}^{n_R} \log\left(1 + \sigma^{-2}\zeta_i\right)$$

Deterministic approximation of the Stieltjes transform

$$I_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \int_{\sigma^2}^{\infty} \left(\frac{1}{\xi} - \mathbb{E}_{\mathbf{P}} m_{\mu}(-\xi)\right) \mathrm{d}\xi$$

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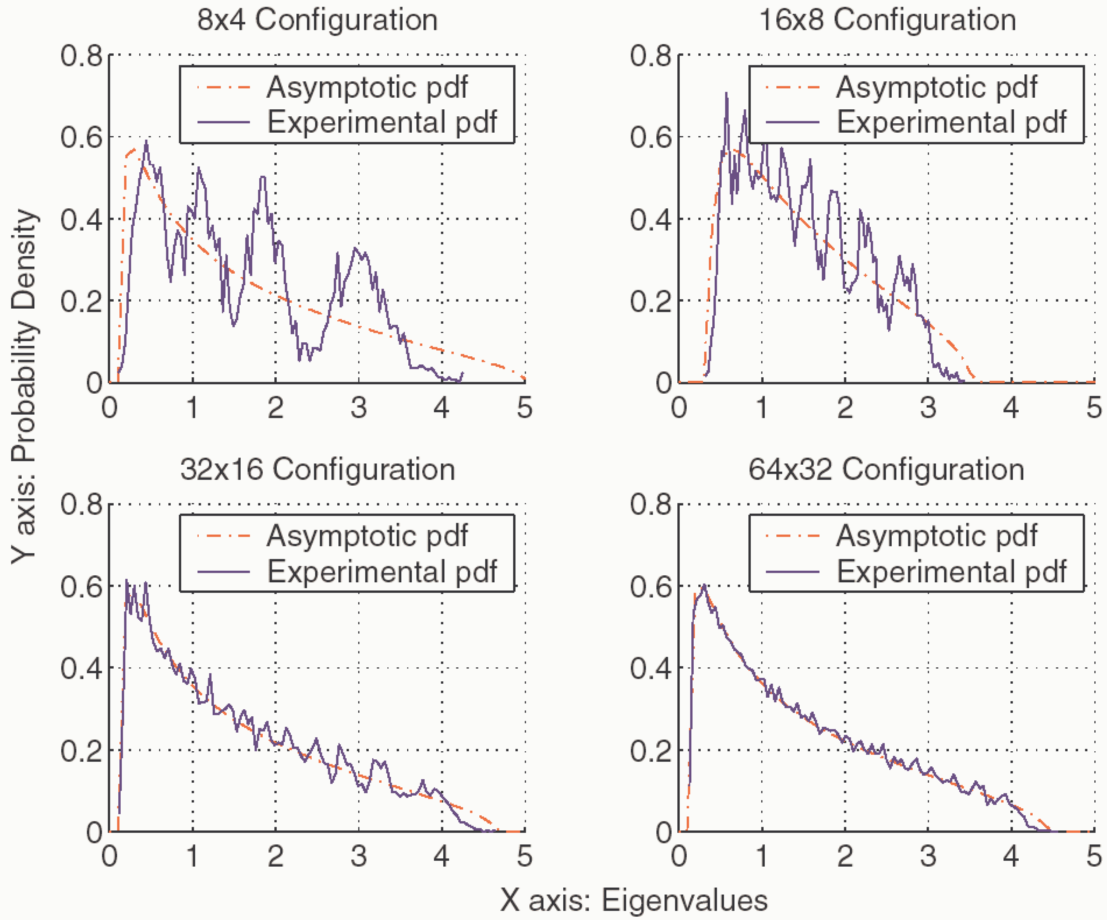
2 - Ergodic approximation with RMT

- Convergence of the **random** and **deterministic** pdfs

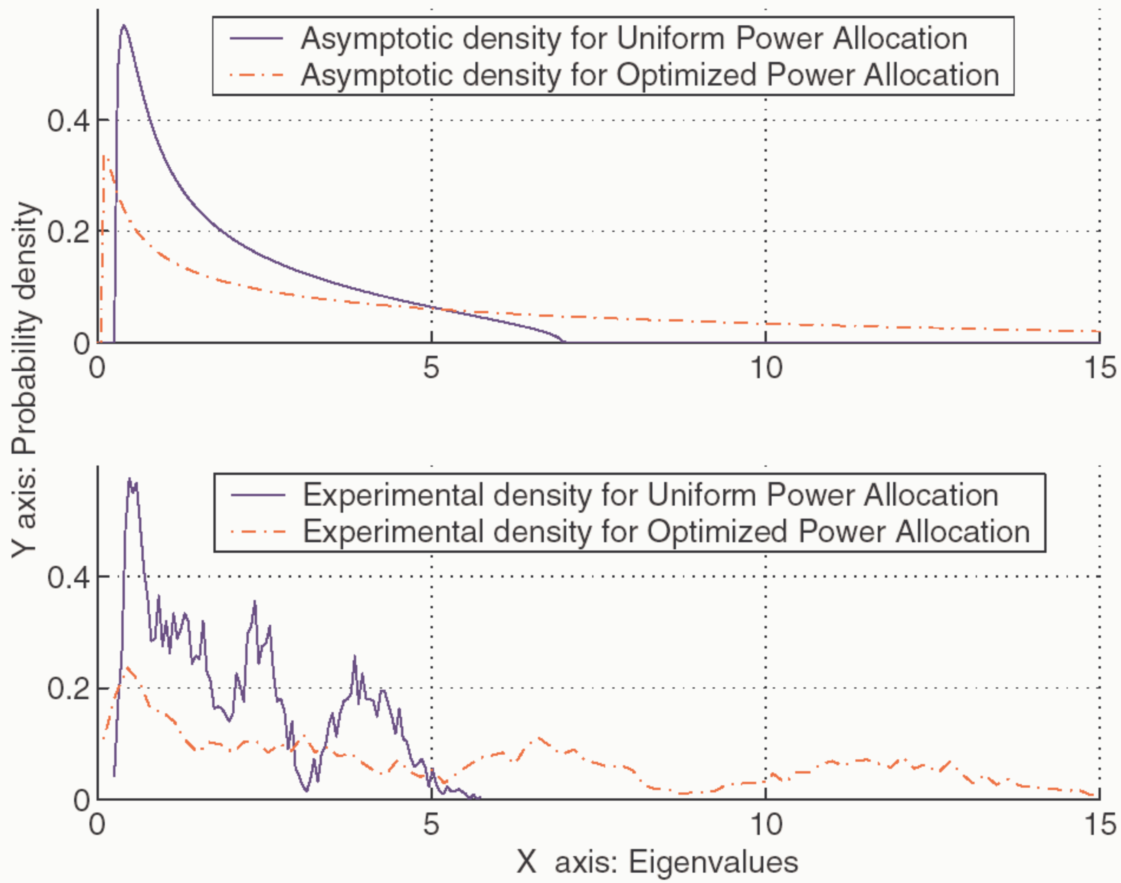


2 - Ergodic approximation with RMT

- Optimization of $C_E = \sup_{\mathbf{\Lambda_Q}} \quad \overline{I}_E(\mathbf{\Lambda_Q}, \mathbf{M}) = \int_{\sigma^2}^{\infty} \left(\frac{1}{\xi} - m_\nu(-\xi) \right) \mathrm{d}\xi$

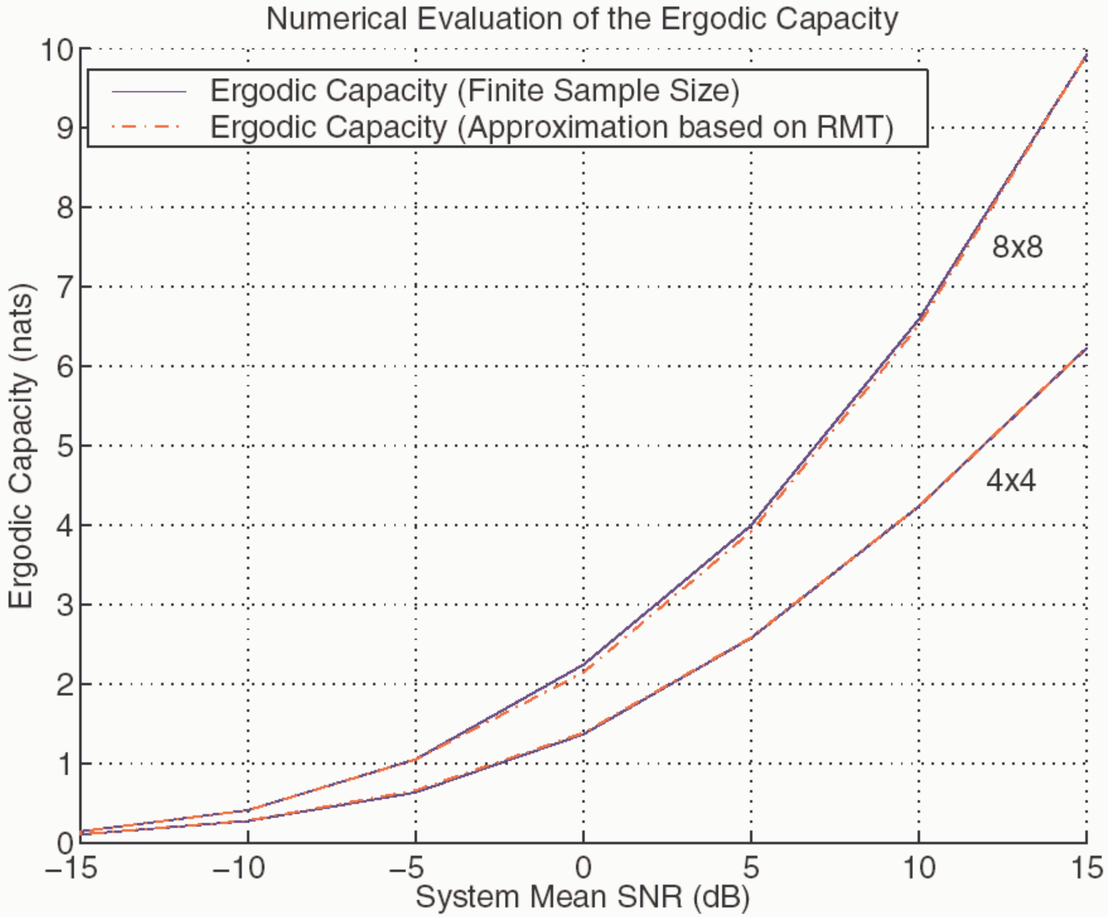
The optimization is done in the **deterministic** approximation →

As a result, the **random** pdf is also optimized →



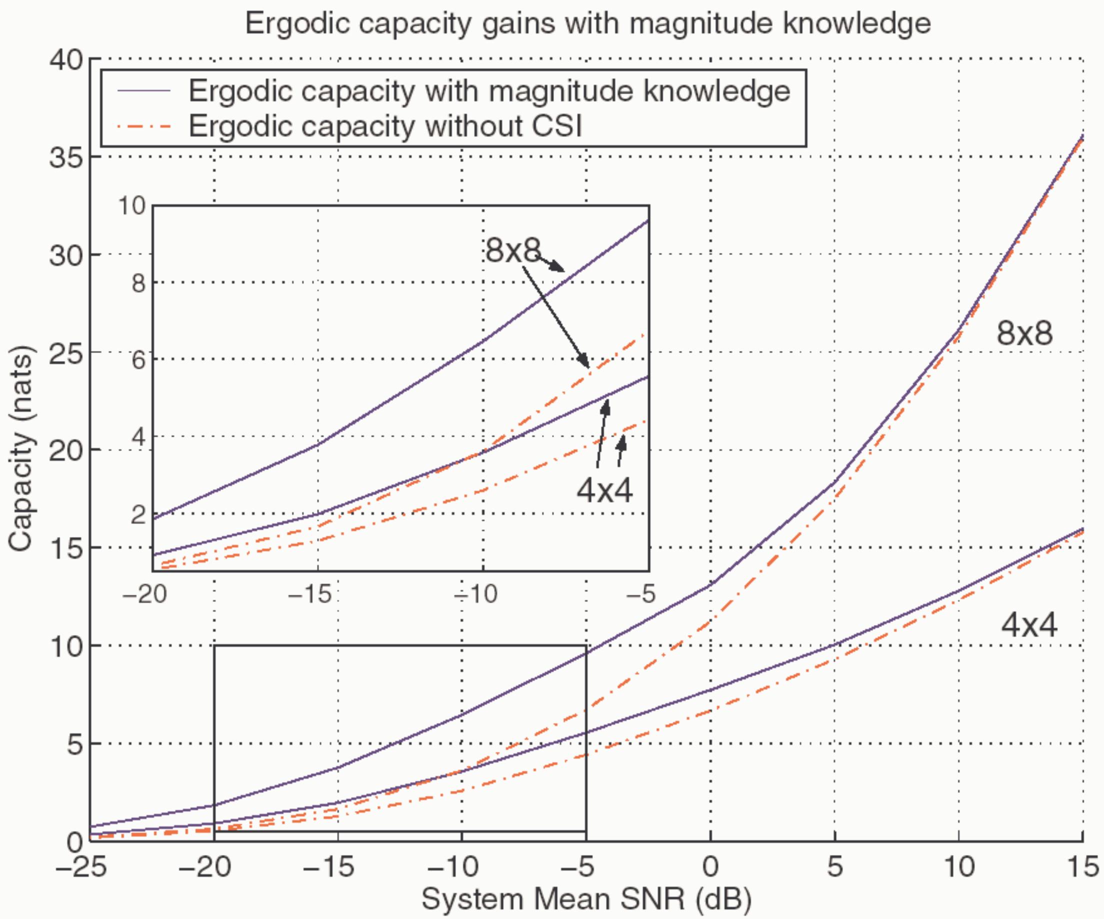
2 - Numerical examples (ergodic)

- Evaluation of the ergodic capacity



2 - Numerical examples (ergodic)

- Value of magnitude knowledge in the ergodic capacity



2 - Compound formulation

- Closed form solution exists for the $2\mathrm{x}n_R$ case
- For the general case the solution has to be calculated numerically

$$I_C(\mathbf{\Lambda_Q}, \mathbf{M}) = \inf_{\mathbf{P} \in \mathcal{P}} \Psi(\mathbf{\Lambda_Q}, \mathbf{M}, \mathbf{P}) \qquad C_C = \sup_{\mathbf{\Lambda_Q}} \quad I_C(\mathbf{\Lambda_Q}, \mathbf{M})$$

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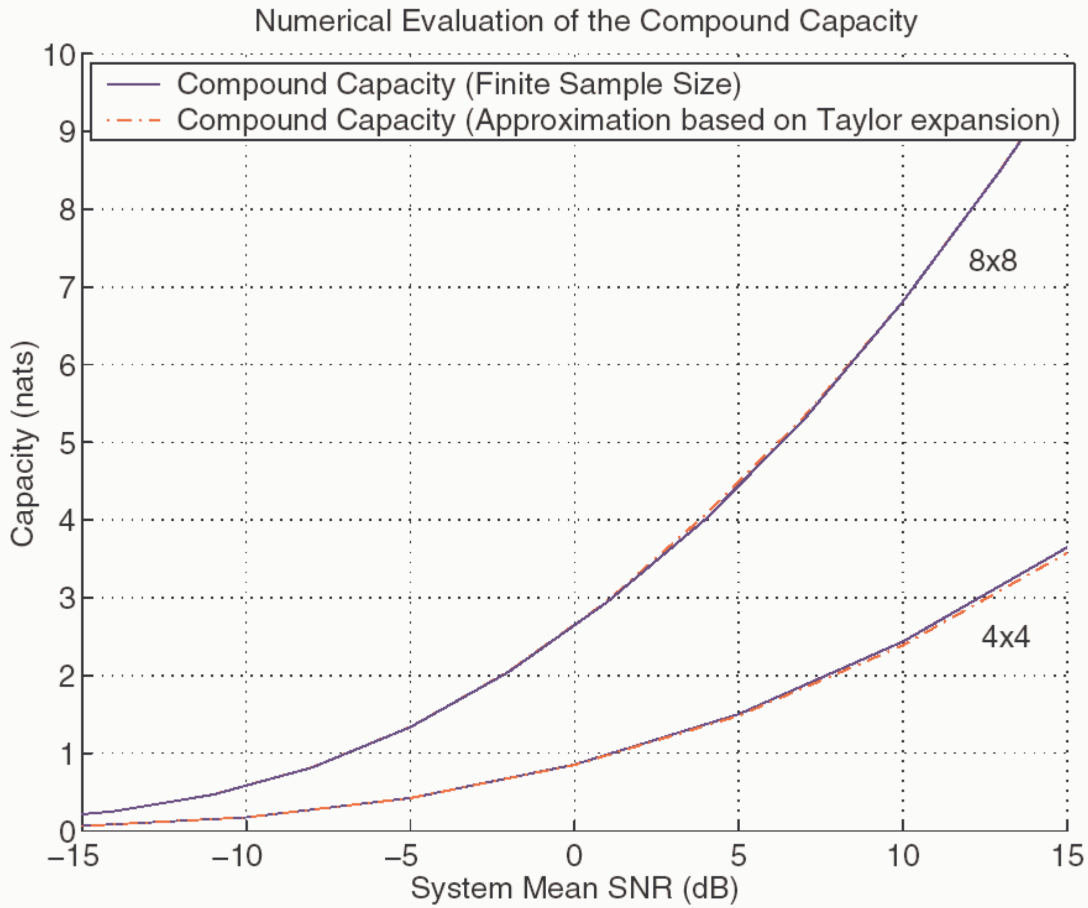
  - Taylor approximation

$$I_C(\mathbf{\Lambda_Q}, \mathbf{M}) \simeq \log\det(\mathbf{I} + \sigma^{-2}\mathbf{M}\mathbf{\Lambda_Q}\mathbf{M}^H)$$

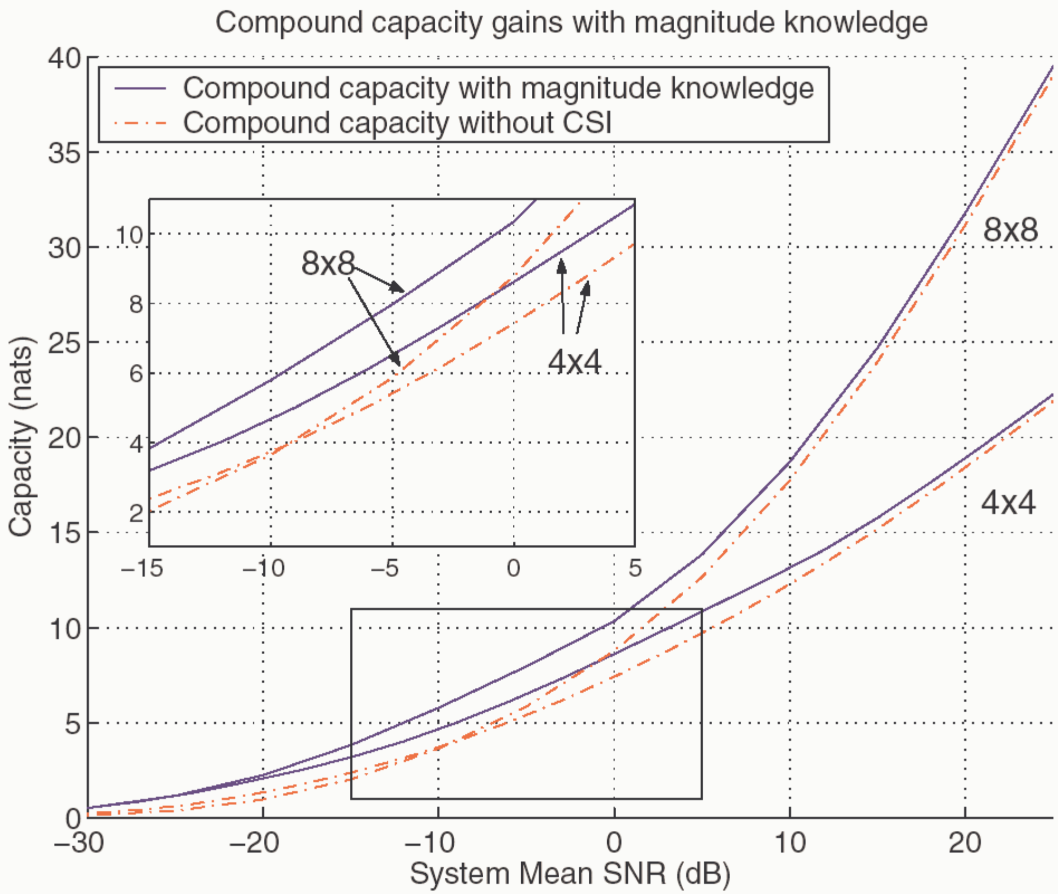
2 - Numerical examples (compound)

- Evaluation of the compound capacity



2 - Numerical examples (compound)

- Value of magnitude knowledge in the compound capacity



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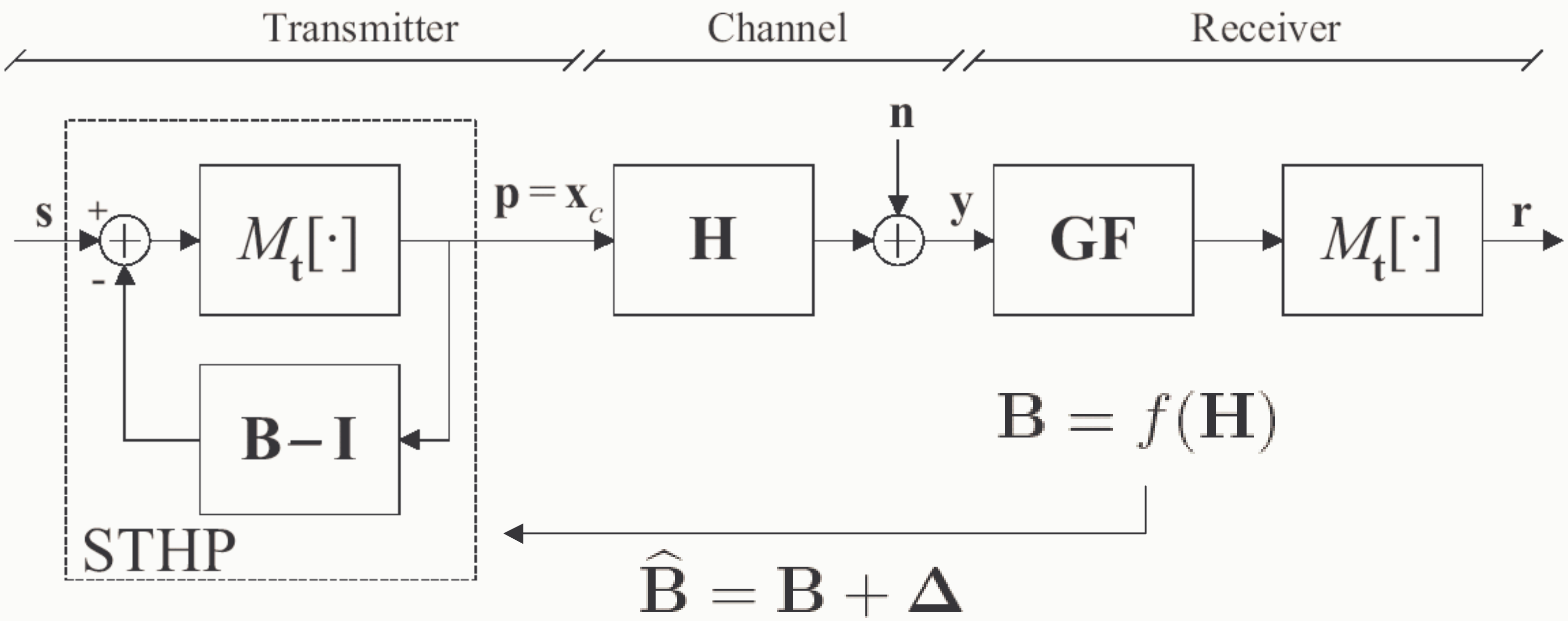
3 - Imperfect CSI

- We study the effects of imperfect CSI on the achievable rates of the Tomlinson-Harashima Precoder (THP)

- The THP is chosen because it is a versatile structure which allows us to deal with the single and multiuser scenarios simultaneously

3 - Tomlinson-Harashima precoder

- For the single user scenario case, it becomes



Contribution:
Allowing different values for the entries of **t**

$$\mathbf{t} = [t_1 \ldots t_N]$$

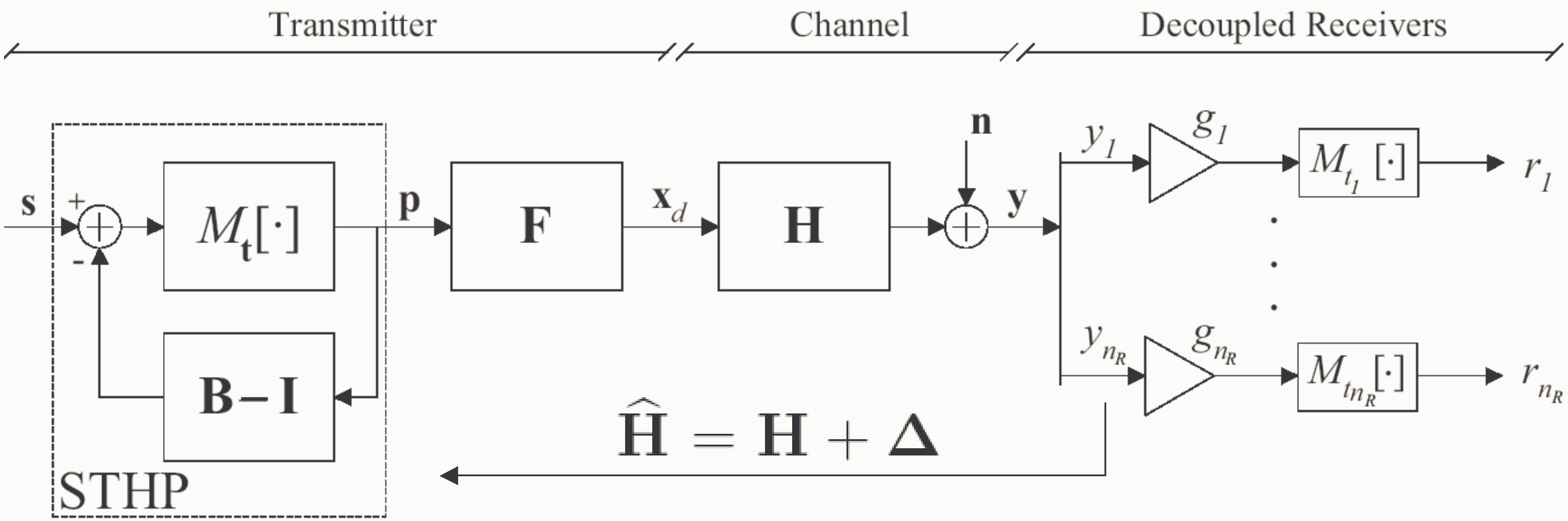
$$P_k^c = 2t_k^2/3$$

$$\mathbf{r} = M_{\mathbf{t}}\left[\mathbf{s} + \mathbf{GFn}\right]$$

$$\mathbf{r}_{\widehat{\mathbf{B}}} = M_{\mathbf{t}}[\mathbf{s} + \mathbf{L}\mathbf{x}_c + \mathbf{G}\tilde{\mathbf{n}}]$$

3 - Tomlinson-Harashima precoder

- For the multiuser scenario case, it becomes



$$\widehat{\mathbf{B}} = f(\widehat{\mathbf{H}}) = \mathbf{B} + \boldsymbol{\Delta}_{\mathbf{B}}$$

$$\widehat{\mathbf{F}} = g(\widehat{\mathbf{H}}) = \mathbf{F} + \boldsymbol{\Delta}_{\mathbf{F}}$$

$$\mathbf{r} = M_{\mathbf{t}}\left[\mathbf{s} + \mathbf{GFn}\right]$$

$$\mathbf{r}_{\widehat{\mathbf{B}},\widehat{\mathbf{F}}} = M_{\mathbf{t}}\left[\mathbf{s} + \boldsymbol{\xi}(\mathbf{s}) + \mathbf{Gn}\right]$$

3 - Achievable rates problem

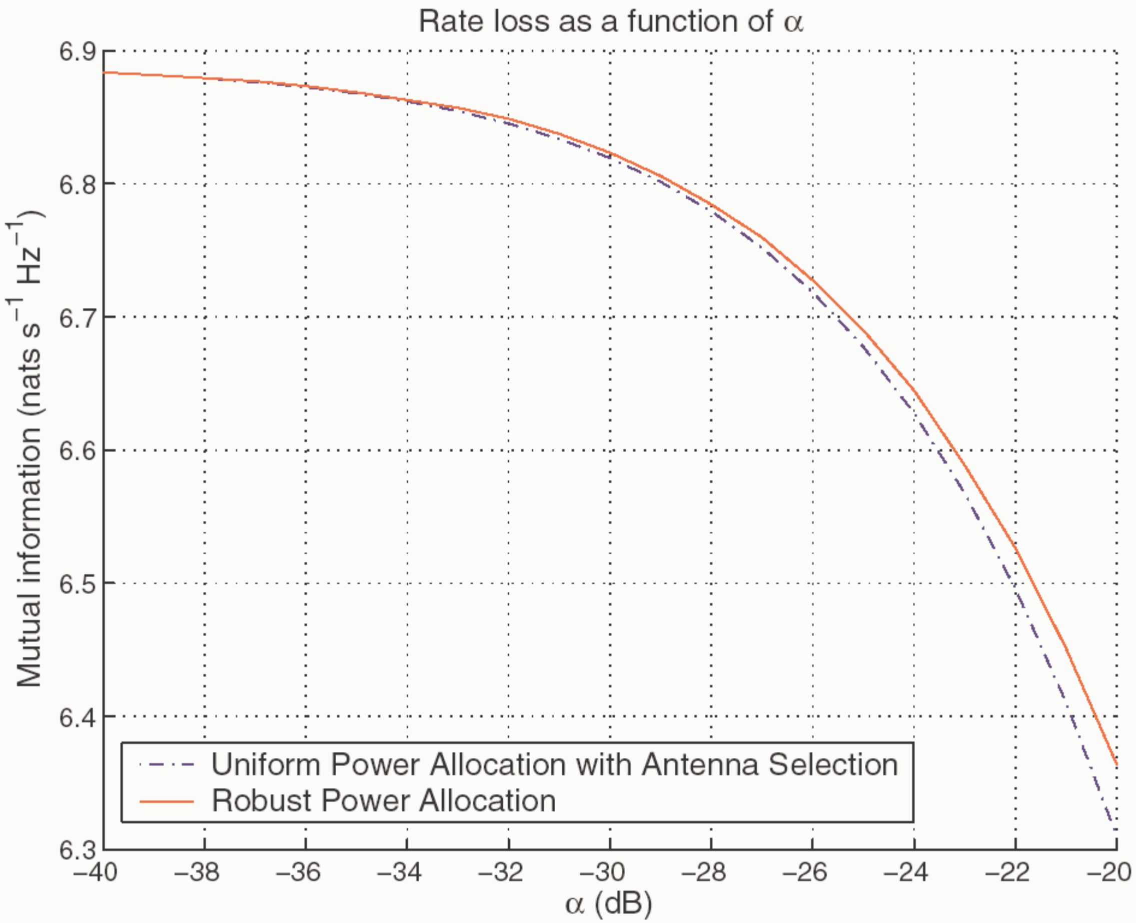
- The problem is to design the modulo operation **t** to optimize the achievable rates (in the multiuser case we optimize the sum rate)

$$
\begin{aligned}
C_{\mathrm{THP}}^{\mathrm{rob}} = \max_{\mathbf{t}} \min_{\boldsymbol{\Delta}} \quad & \sum_{k=1}^{N} I(s_k; r_k) \\
\text{s.t.} \quad & \sum_{k=1}^{N} \frac{2t_k^2}{3} \leq P_T \\
& \boldsymbol{\Delta} \in \mathcal{R}
\end{aligned}
$$

- The solution has to be calculated numerically
- Introducing some approximations the solution can be found in closed form

3 - Solution in the single user case

- Mutual information loss comparison



Red curve

Robust design of **t**

Blue curve

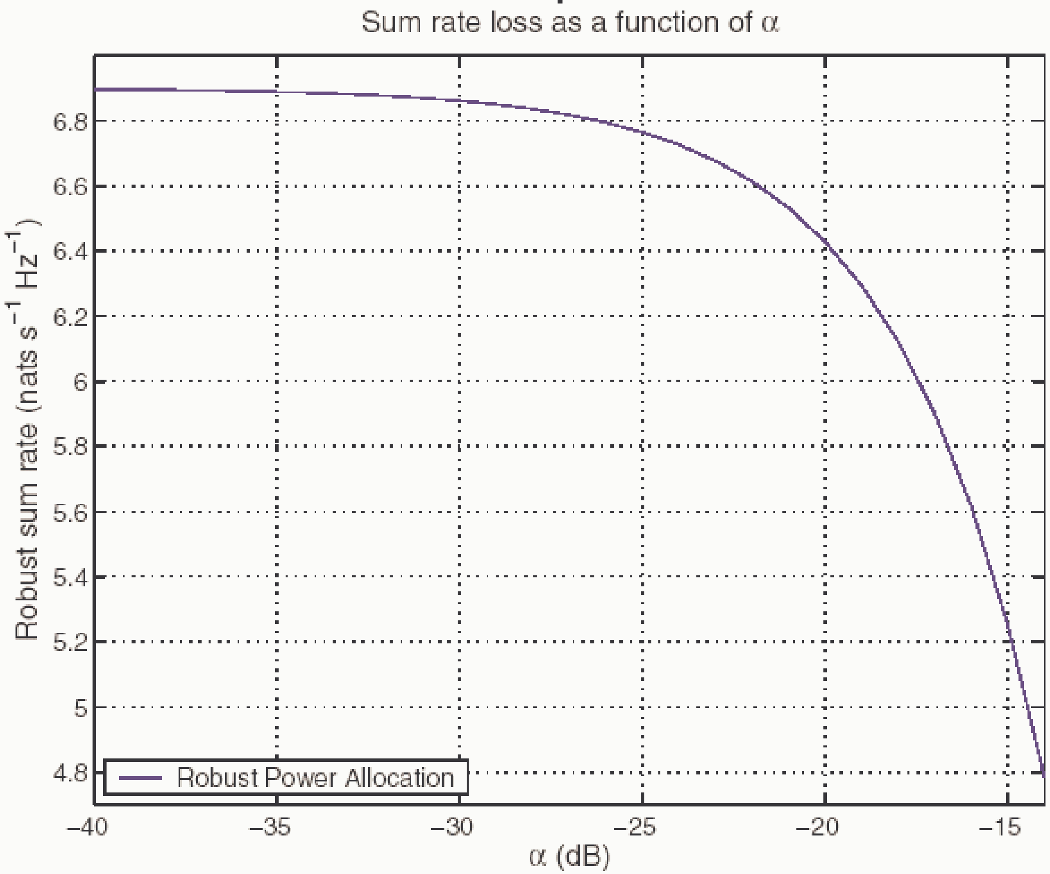
$$t_k = \sqrt{3P_T/(2|\mathcal{G}|)}$$

or

$$t_k = 0$$

3 - Solution in the multiuser case

- The (approximate) solution is user selection and uniform modulo operation



Blue curve

$$t_k = \sqrt{3P_T/(2|\mathcal{G}|)}$$

or

$$t_k = 0$$

Outline

- Communications set-up
- Cases of study
  - Perfect CSI and single user communications
  - Incomplete CSI and single user communications
  - Imperfect CSI, from single to multiuser communications
  - Imperfect CSI and multiuser communications
- Conclusions

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4 - Capacity results

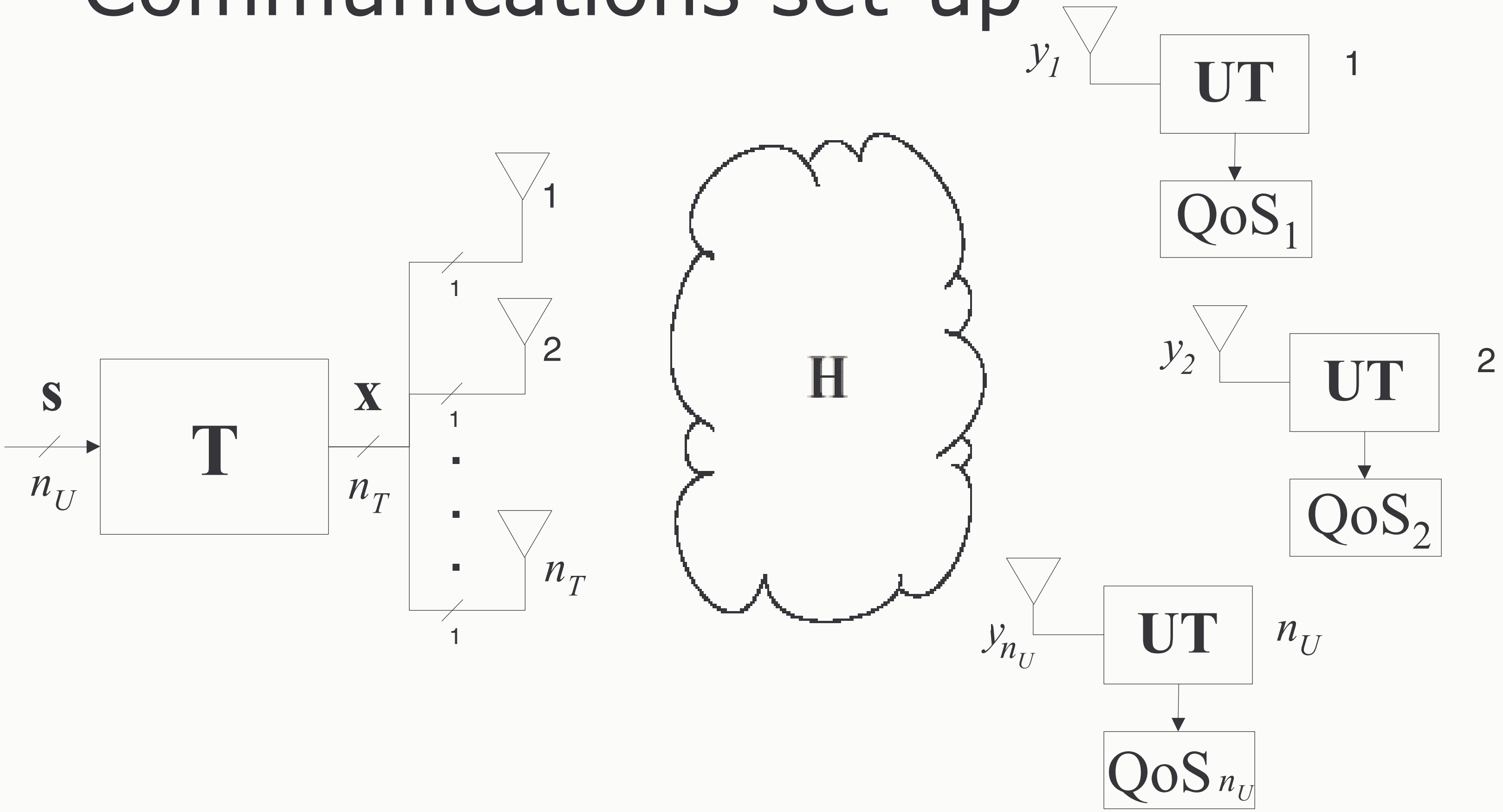
- The capacity region for the multiuser (broadcast) linear MIMO channel with Gaussian noise has been recently found [Weingarten, Steinberg, Shamai 04-06].
- For the imperfect CSI case almost no results exist
- We focus on practical transmission schemes
  - The downlink problem

4 - The downlink problem

- Design a linear transmitter for the broadcast channel such that:
  - Each user has a QoS constraint
  - The objective is to transmit the minimum power

$$\begin{aligned} &\text{minimize} \quad P_T \\ &\text{subject to} \quad \mathsf{qos}_i \geq \mathsf{qos}_i^0, \quad \forall i \in \{1, 2, \ldots, n_U\}. \end{aligned}$$

4 - Communications set-up



$$\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_{n_U}]^H \qquad y_i = [\mathbf{y}]_i = \mathbf{h}_i^H \mathbf{x} + n_i,$$

4 - The solution

- The downlink problem has been studied and solved by two research groups (Bengtsson and Boche)
- Both groups considered that:
  - The QoS indicator is the SINR
  - Perfect CSI is available at the transmitter and receiver
- For the imperfect CSI case, this problem has been addressed and solved by two research groups (Gershman and Bengtsson)
  - The uncertainty is in the correlation matrix

$$\tilde{\mathbf{R}}_{\mathbf{H}} = \mathbf{R}_{\mathbf{H}} + \boldsymbol{\Delta}$$

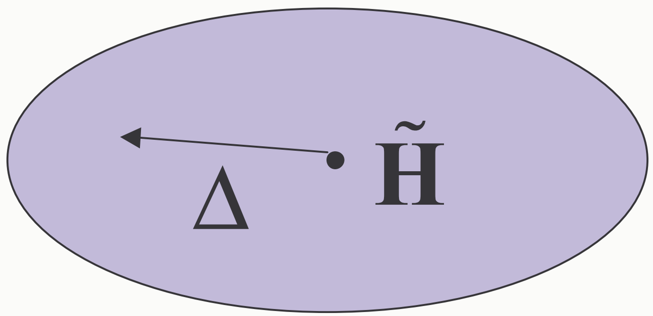
4 - Our problem

- We want to solve the downlink problem for imperfect CSI in the channel estimate

$$\mathbf{H} = \tilde{\mathbf{H}} + \boldsymbol{\Delta}$$

- We assume that the actual channel belongs to an uncertainty region around the channel estimate

$$\boldsymbol{\Delta} \in \mathcal{R}$$



4 - Problem statement

- The problem becomes

$$\begin{aligned} &\text{minimize} \quad P_T \\ &\text{subject to} \quad \mathsf{qos}_i \geq \mathsf{qos}_i^0, \quad \forall i \in \{1, 2, \dots, n_U\}, \\ &\qquad\qquad\qquad\qquad\qquad\quad \forall \boldsymbol{\Delta} \in \mathcal{R}. \end{aligned}$$

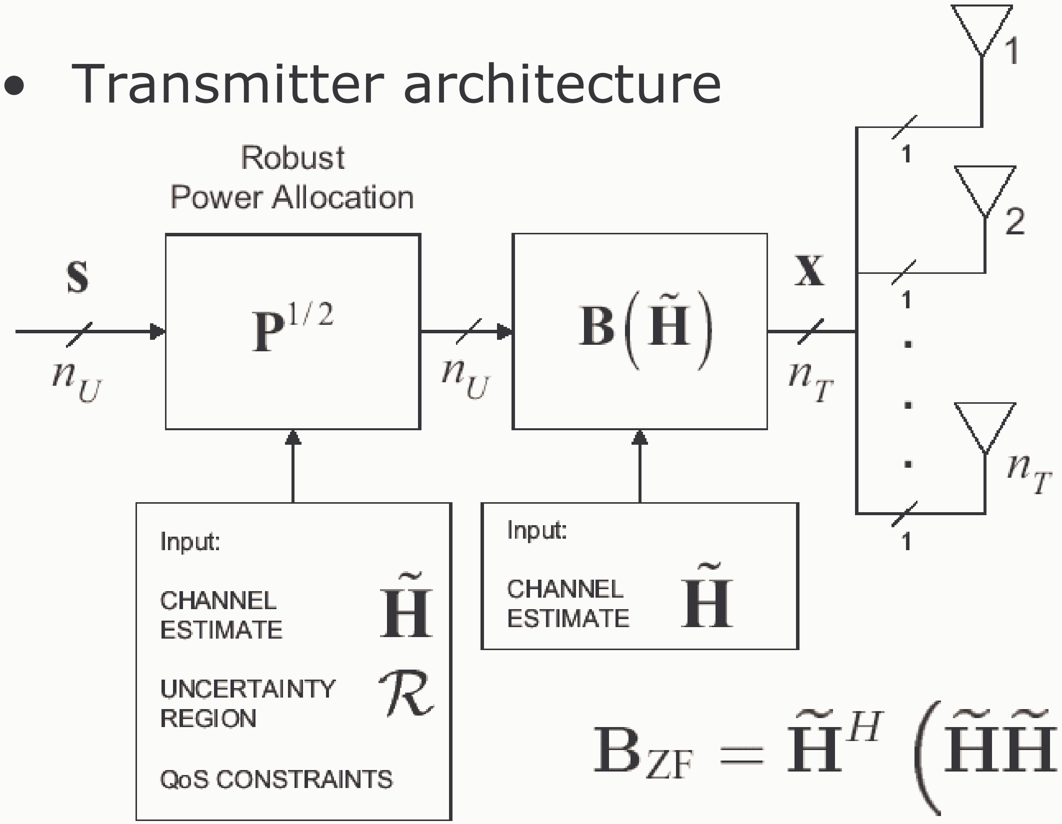
- To be able to find a solution we have to simplify the architecture:
  - The linear transmitter is divided in two parts and only one of them is a design parameter

$$\mathbf{T} = \mathbf{B}\mathbf{P}^{1/2}$$

  - The receivers estimate the symbol by a simple division

4 - Simplification at the transmitter

- Transmitter architecture



$$P_T(\mathbf{P}) = \operatorname{Tr} \mathbf{B}\mathbf{P}\mathbf{B}^H$$

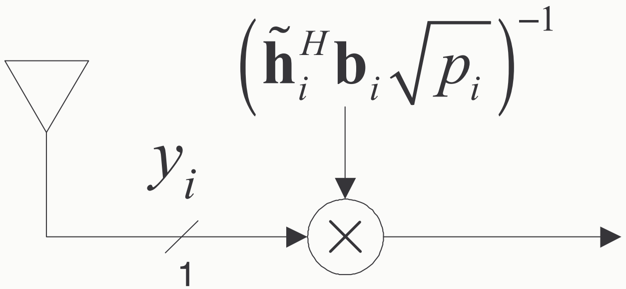
$$\mathbf{B}_{\mathrm{ZF}} = \tilde{\mathbf{H}}^H \left(\tilde{\mathbf{H}}\tilde{\mathbf{H}}^H\right)^{-1}$$

$$\mathbf{B}_{\mathrm{WF}} = \left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \alpha\mathbf{I}\right)^{-1}\tilde{\mathbf{H}}$$

$$\mathbf{B} = \mathbf{I}$$

4 - Receivers simplification

- Receiver architecture



$$\hat{s}_i = s_i + \frac{(\tilde{\mathbf{h}}_i^H \bar{\mathbf{B}}_i + \boldsymbol{\delta}_i^H \mathbf{B})\mathbf{P}^{1/2}\mathbf{s} + n_i}{\tilde{\mathbf{h}}_i^H \mathbf{b}_i \sqrt{p_i}}$$

- The QoS indicator is chosen to be the inverse of the MSE (effective SINR)

$$\text{mse}_i = \mathbb{E}|s_i - \hat{s}_i|^2 \qquad \frac{1}{\text{mse}_i} \triangleq \text{esinr}_i.$$

4 - Formal problem statement

- The problem can then be expressed as

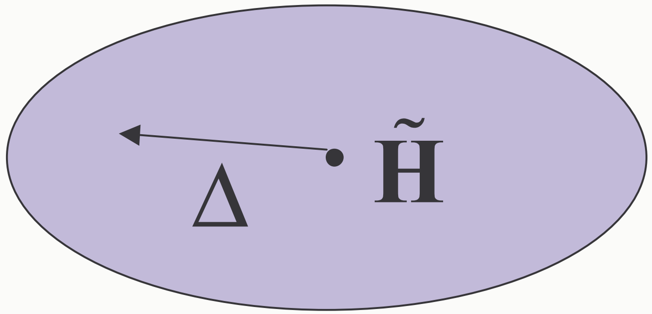
$$\begin{aligned} \underset{\mathbf{P}}{\text{minimize}} \quad & P_T = \operatorname{Tr} \mathbf{BPB}^H \\ \text{subject to} \quad & \mathsf{esinr}_i(\mathbf{\Delta}, \mathbf{P}) \geq \mathsf{esinr}_i^0, \quad \forall i \in \{1, 2, \ldots, n_U\}, \\ & \qquad\qquad\qquad\qquad\qquad \forall \mathbf{\Delta} \in \mathcal{R}. \end{aligned}$$

- We managed to reformulate the problem in convex form no matter what is the shape of the uncertainty region

- To go further we need to define:
  - The linear transmitter matrix: $\mathbf{B}_{ZF}$
  - The shape and size of the uncertainty region

4 - The uncertainty region

$$\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{\Delta}$$

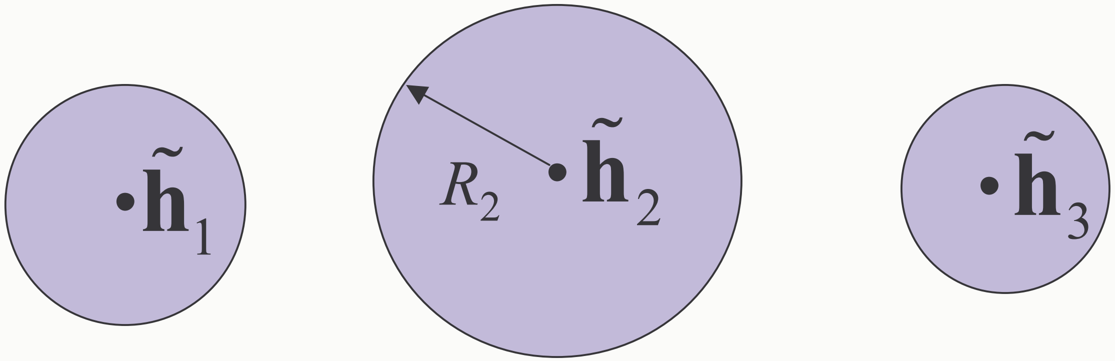


$$\mathbf{\Delta} \in \mathcal{R}$$

- The shape and size of the uncertainty region must be connected with the physical phenomenon producing the error
  - Estimation Gaussian noise
  - Quantization effects
  - Combinations of both

4 - Estimation Gaussian noise

- Each user estimates its own channel and feeds the estimation back to the transmitter (assumed error free)
- The estimate is a version of the actual channel corrupted with AWGN
- The uncertainty region is a set of spherical regions, centered at the estimate of the channel of each user



- Since the error is unbounded there is a certain probability that the actual channel is outside the uncertainty region (outage event declared)

4 - Estimation Gaussian noise

- For this particular case a closed form solution exists (derived from the KKT)

$$p_i^\star = \mathsf{esinr}_i^0 \sigma^2 + \mathsf{esinr}_i^0 \mu R_i^2 = \mathsf{esinr}_i^0 \left(R_i^2 \mu + \sigma^2 \right)$$

- The μ parameter is the solution to a fixed point equation (can be calculated numerically in a very efficient way)

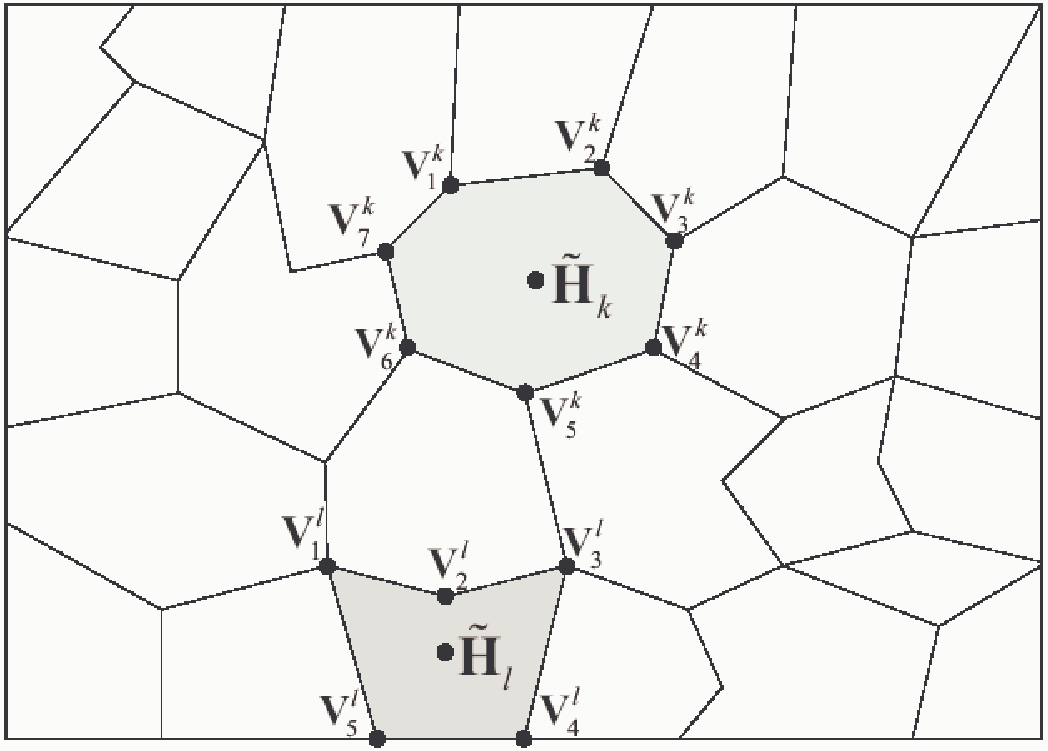
- The power allocation with perfect CSI is given by

$$p_i^{\star,\mathrm{perfect}} = \mathsf{esinr}_i^0 \sigma^2$$

- The price of the robust design is $\mathsf{esinr}_i^0 \mu R_i^2$

4 - Quantization effects

- Each user quantizes the actual realization of the channel and feeds it back to the transmitter (possibly through a digital link)



$$\tilde{\mathbf{h}}_i = \mathcal{Q}\{\mathbf{h}_i\}$$

4 - Quantization effects

- In this case the convex optimization problem can be simplified to a linear program
  - Linear objective
  - Linear constraints
  - In general, no closed form is available

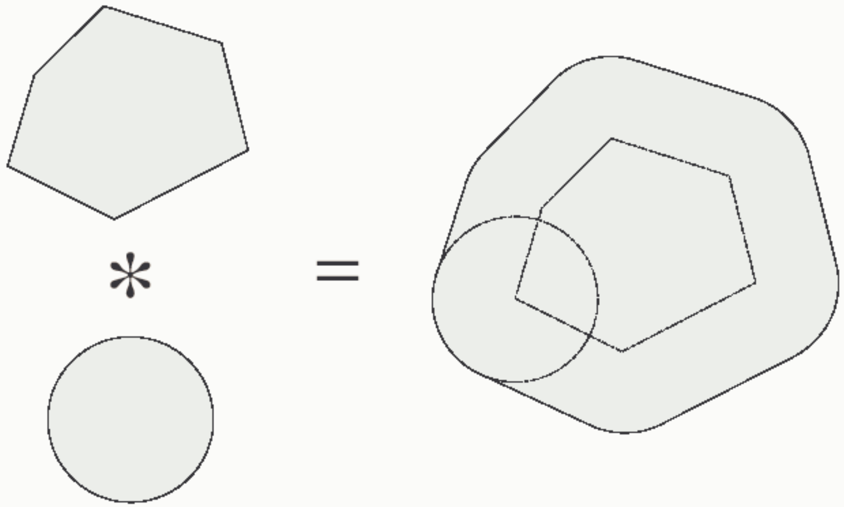
- There are powerful tools to solve this kind of problems

$$\begin{aligned} \underset{\mathbf{P}}{\text{minimize}} \quad & \operatorname{Tr} \mathbf{B}_{\text{ZF}} \mathbf{P} \mathbf{B}_{\text{ZF}}^{H}, \\ \text{subject to} \quad & \mathbf{v}_{m,i}^{H} \mathbf{B}_{\text{ZF}} \mathbf{P} \mathbf{B}_{\text{ZF}}^{H} \mathbf{v}_{m,i} - \mathsf{mse}_{i}^{0} p_i + \sigma^2 \leq 0, \quad \forall i \end{aligned}$$

4 - Combination of regions

- In the most general setup each user
  1. Estimates the channel
  2. Quantizes this estimate

- In this case, the uncertainty region becomes more involved



4 - Combination of regions

- The problem is still convex, but it doesn't have a particular structure

$$\begin{aligned}
&\underset{\mathbf{P}}{\text{minimize}} \quad \operatorname{Tr} \mathbf{B}_{\mathrm{ZF}} \mathbf{P} \mathbf{B}_{\mathrm{ZF}}^{H}, \\
&\text{subject to} \quad \left(\mathbf{s}_{m,i}^{\star}(\mathbf{P}) + \mathbf{v}_{m,i}\right)^{H} \mathbf{B}_{\mathrm{ZF}} \mathbf{P} \mathbf{B}_{\mathrm{ZF}}^{H} \left(\mathbf{s}_{m,i}^{\star}(\mathbf{P}) + \mathbf{v}_{m,i}\right) - \mathsf{mse}_{i}^{0} p_{i} + \sigma^{2} \leq 0,
\end{aligned}$$

- Numerical methods are applied to compute the solution

- The computational load is in the calculation of the restriction

4 - Feasibility test

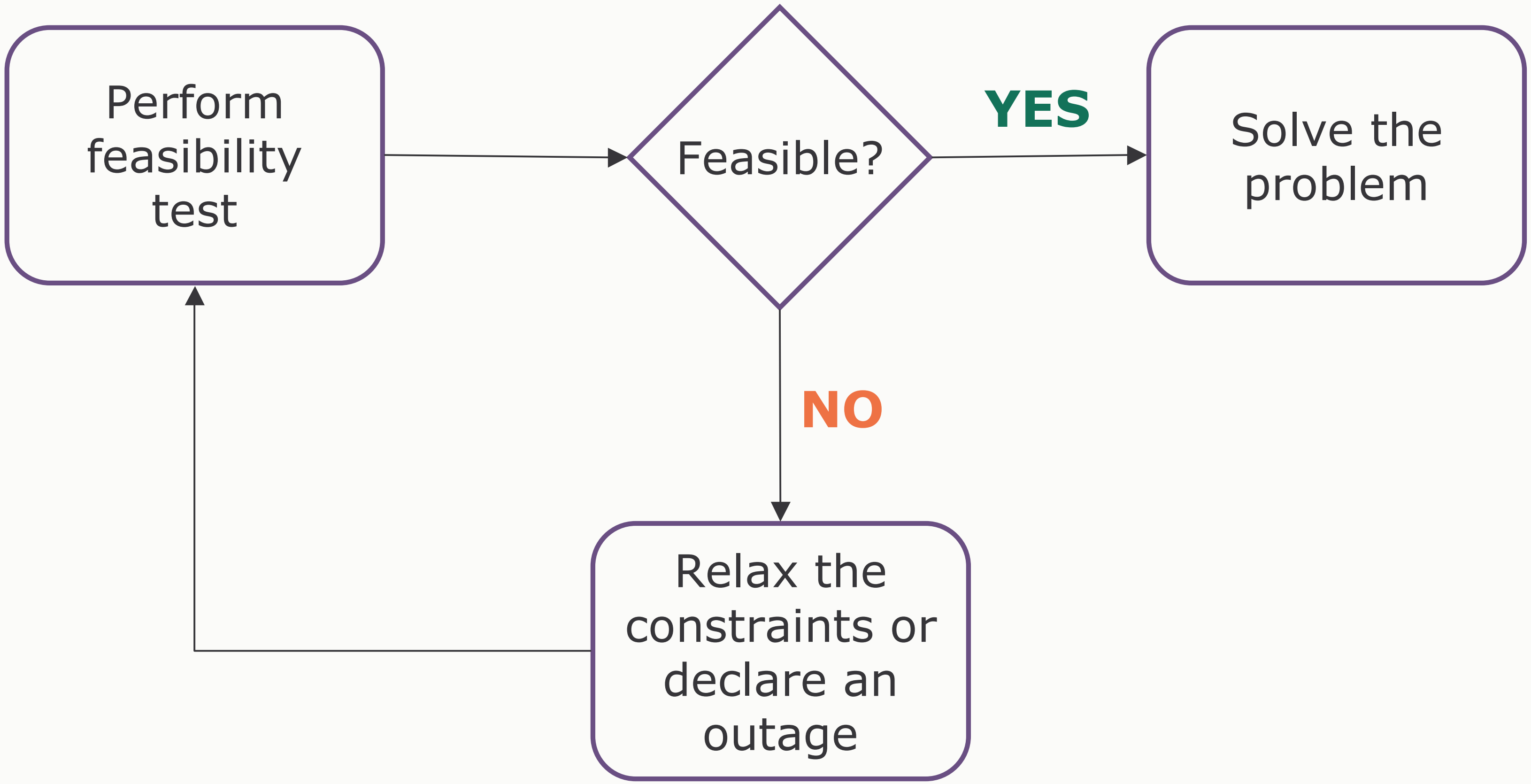
- The problem

$$\begin{aligned} \underset{\mathbf{P}}{\text{minimize}} \quad & P_T = \operatorname{Tr} \mathbf{B}\mathbf{P}\mathbf{B}^H \\ \text{subject to} \quad & \mathsf{esinr}_i(\boldsymbol{\Delta}, \mathbf{P}) \geq \mathsf{esinr}_i^0, \quad \forall i \in \{1, 2, \ldots, n_U\}, \\ & \qquad\qquad\qquad\qquad\qquad \forall \boldsymbol{\Delta} \in \mathcal{R}. \end{aligned}$$

  may become infeasible if there exists no **P** matrix such that the constraints can be simultaneously fulfilled

- We derived a feasibility check in the form of a convex problem, which should be solved previously

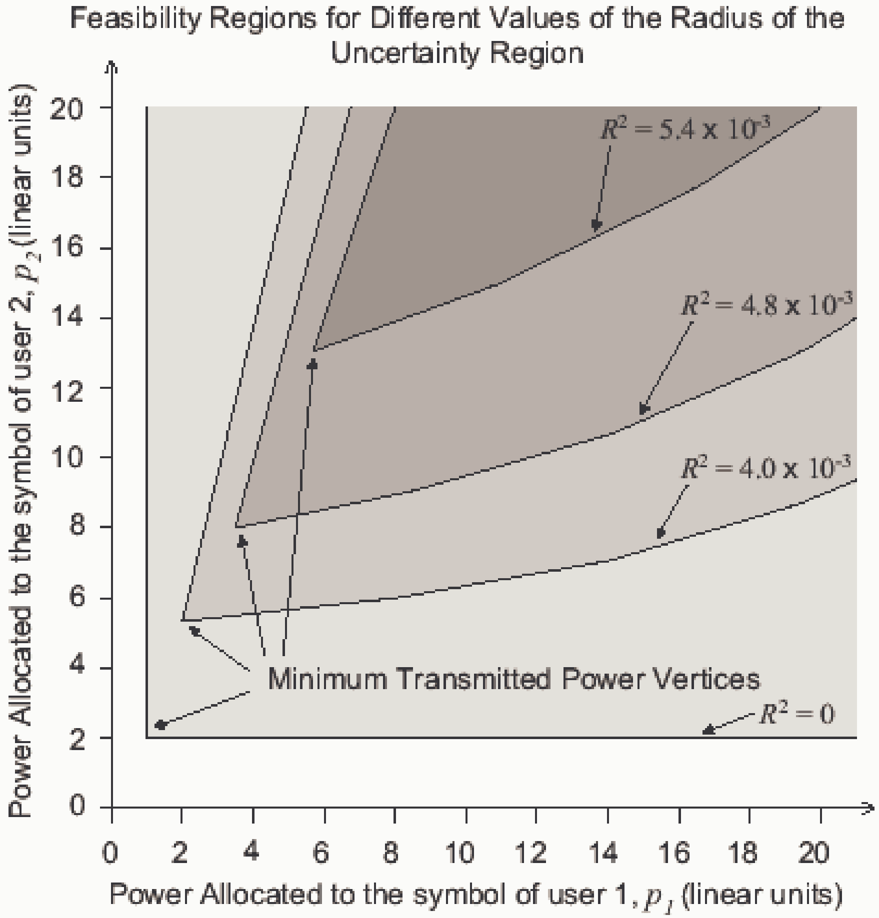
4 - Problem solution algorithm



4 - Numerical examples

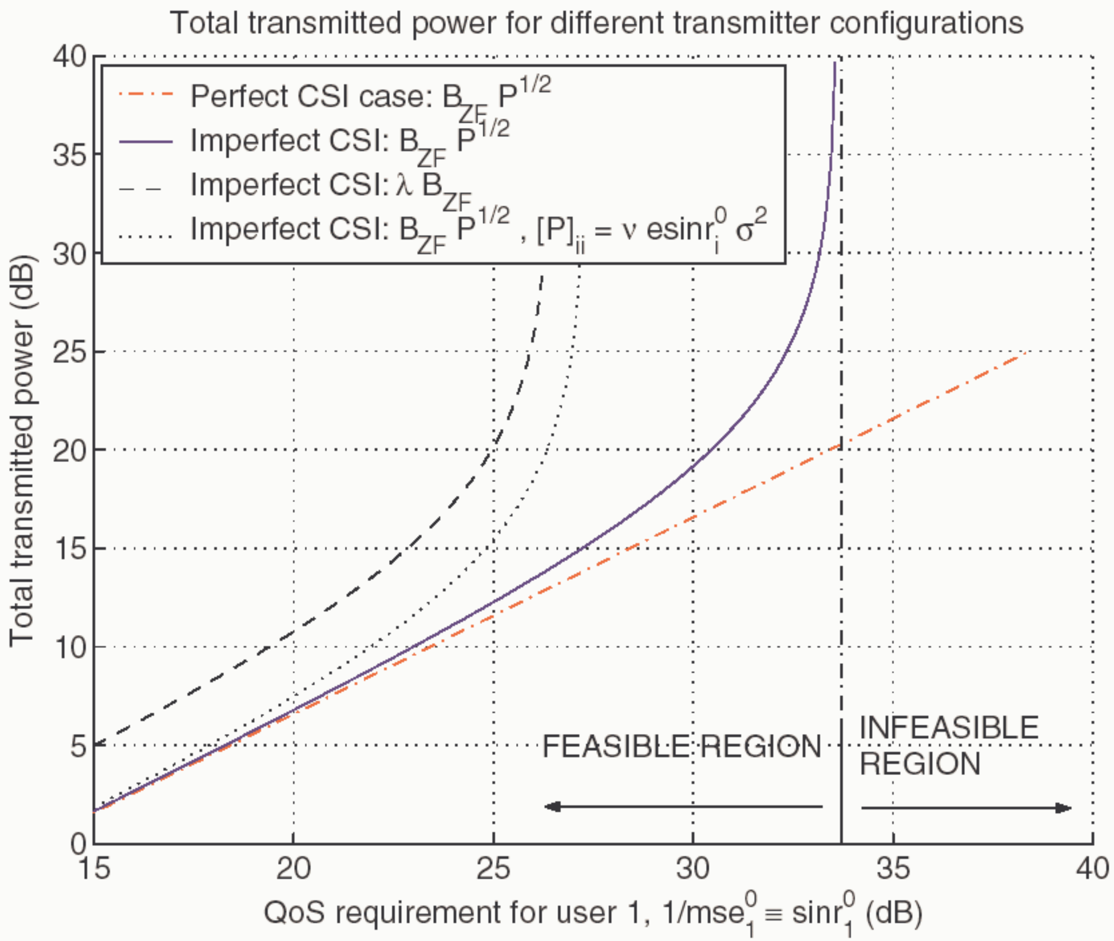
- Feasibility region

2 user scenario



4 - Numerical examples

- Comparison for different power allocation policies



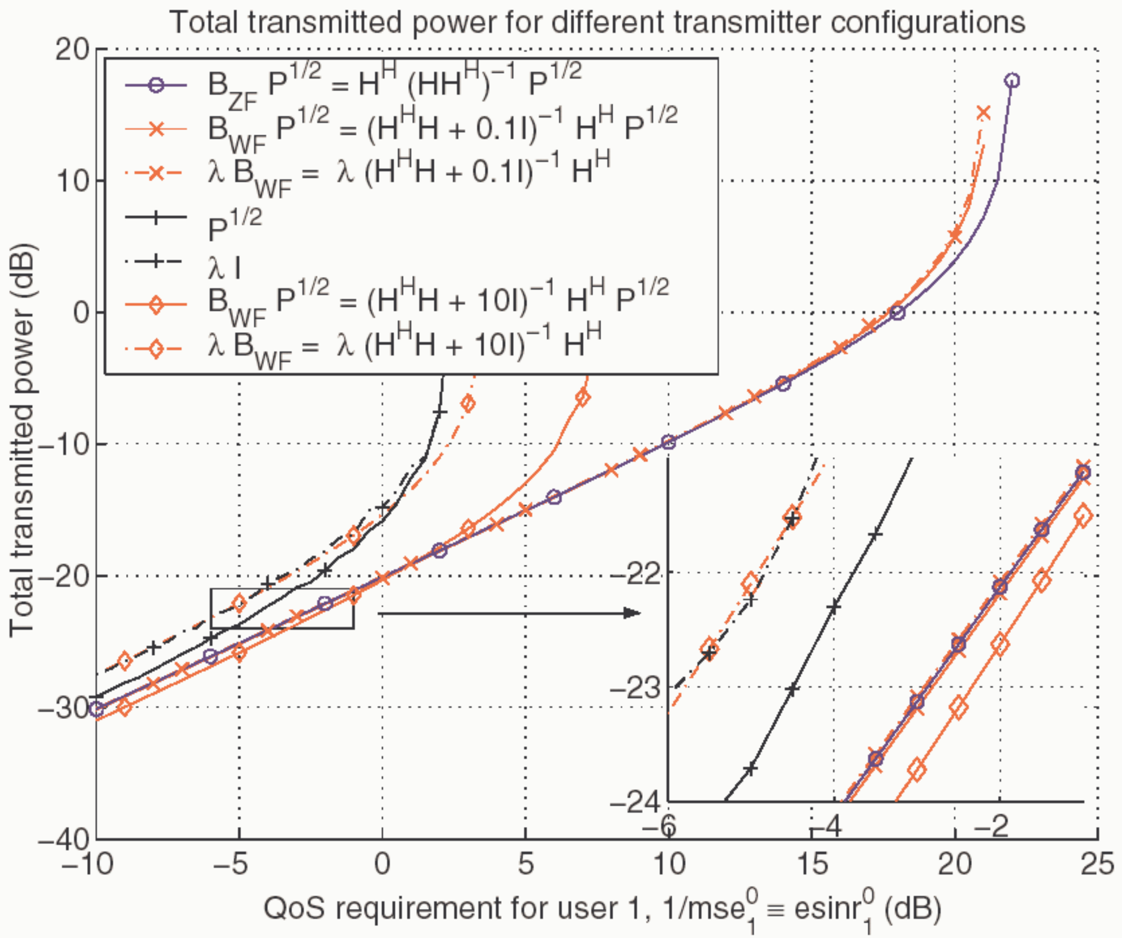
$$\mathbf{B} = \mathbf{B}_{\mathrm{ZF}}$$

8 user scenario

4 - Numerical examples

- Comparison for different choice of transmitter matrix **B**

8 user scenario



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Conclusions

- The impact of the CSI on the design and analysis of multi-antenna systems has been characterized
- Perfect CSI (single user)
  - Design of linear transmitter for practical constellations
- Incomplete CSI (single user)
  - Analysis of the performance limits for the case of magnitude knowledge and phase uncertainty
- Imperfect CSI (single and multiuser)
  - Analysis of the achievable rates of the Tomlinson-Harashima precoder in single and multiuser scenarios
- Imperfect CSI (multiuser)
  - Design of a robust linear transmitter to minimize the transmitted power while guaranteeing QoS to the users

Thank you very much!