Impact of channel state information on the analysis and design of multi-antenna communication systems

(PhD dissertation)

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General overview

• Objectives:
  – Analysis and design of multi-antenna systems
    • Analysis in terms of achievable rates and capacity (ultimate performance limits)
    • Design carried out according to practical criteria
      – Mean squared error
      – Received symbols distance
  – Characterize the impact of the CSI (Available information on the channel state)
Outline

- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)
- Conclusions (Ch. 7)
Outline

• Communications set-up (Ch. 2)

• Cases of study (Ch. 3 -- 6)

  Most simple case: single user communications with perfect CSI

  –**Contribution:** DESIGN of a linear transmitter to maximize the minimum distance among the received constellation points

• Conclusions (Ch. 7)
Outline

- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)
  - Perfect CSI and single user communications
- Conclusions (Ch. 7)
Assuming perfect CSI is rather unrealistic (specially in practical deployments)
We studied communication systems with power feedback
This evolved into capacity analysis with magnitude knowledge and phase uncertainty

- Contribution: ANALYSIS of the capacity for this kind of CSI
Outline

- Communications set-up (Ch. 2)
- Cases of study (Ch. 3 -- 6)
  - Perfect CSI and single user communications
  - Incomplete CSI and single user communications
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• Communications set-up (Ch. 2)

• Cases of study (Ch. 3 -- 6)

  One of the main advantages of multi-antenna systems is their multiuser capabilities.
  We decided to analyze the achievable rates of the THP because it unifies the single and multiuser formulations.

  -- **Contribution**: **ANALYSIS** of the achievable rates for THP in the presence of errors in the CSI

• Conclusions (Ch. 7)
Outline

• Communications set-up (Ch. 2)

• Cases of study (Ch. 3 -- 6)
  – Perfect CSI and single user communications
  – Incomplete CSI and single user communications
  – Imperfect CSI, from single to multiuser communications

• Conclusions (Ch. 7)
The results were disappointing, because they did not represent a great enhancement with respect to the existing literature. It seemed that THP was a limiting architecture. Fully focus on the design of a linear transmitter for a multiuser system with imperfect CSI.

- *Contribution*: **DESIGN** of a robust linear transmitter to guarantee QoS requirement and transmit the minimum power.
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  - Imperfect CSI and multiuser communications
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• Conclusions
Communications set-up

- Single user scenario

\[
\begin{align*}
\omega & \xrightarrow{\text{ENCODER}} x^N & p(y^N|x^N) & \xrightarrow{\text{DECODER}} \hat{\omega} \\
& \xrightarrow{\text{CHANNEL STATE}} & & \\
& & \xrightarrow{\text{CHANNEL}} & \\
& \xrightarrow{\text{RECEIVER}} & y^N & \\
& & & \xrightarrow{\text{TRANSMITTER}} & \\
\end{align*}
\]

- Signal transmitted through each one of the antennas
- Signal received through each one of the antennas
Communications set-up

- Multiuser scenario (broadcast)
Communications set-up

- Linear transmitter scenario
Communications set-up

- The multiple-antenna channel (MIMO)

  - Memoryless \[ p(y^N|x^N) = \prod_{i=1}^{N} p(y_i|x_i) \]

  - Linear \[ y = Hx + n \]

  - Corrupted with additive white Gaussian noise

\[ n \sim CN(0, R_n) \quad E[n] = 0 \quad E[nn^H] = R_n \]
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• Conclusions
Cases of study

- Different degrees of CSI

- **Perfect CSI**
  - Optimal linear filtering

- **Imperfect CSI**
  - Robust designs

- **Incomplete CSI**
  - Statistical design
  - Maximin design

- **No CSI**
  - Space-time codes
Cases of study

- Different degrees of CSI

**Perfect CSI**
Optimal linear filtering

**Imperfect CSI**
Robust designs

**Incomplete CSI**
Statistical design
Maximin design

**No CSI**
Space-time codes
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• Conclusions
1 - Capacity results (by Telatar)

- The transmitter can be matched to each channel realization
  \[ C = \max_Q \log \det \left( I + R_n^{-1}HQH^H \right) \quad Q = E_{xx}^H \]

- The optimal \( Q \) is obtained from the SVD decomposition

\[ Q^* = U_H \Lambda_Q U_H^H \]

where

\[ \lambda_{Q,j}^* = \left( \mu - \frac{1}{\lambda_{H,j}} \right)^+ \]
1 - Capacity results

- Since the optimal covariance matrix is a capacity achieving structure is

\[ Q^* = U_H \Lambda_Q U_H^H \]

\[ B^* = U_H \Lambda_Q^{1/2} V^H \]

- The capacity achieving codes are practically unfeasible
  - Infinite length
  - Gaussian distributed
1 - Practical transmitter design

- We want to solve the problem for the case where
  - We are transmitting a given constellation (BPSK, QPSK, ...)
  - The detection is performed in a symbol by symbol basis

\[
\hat{s} = \arg \max_{s \in \mathcal{C}} \log \mathcal{L}(y, Bs)
\]

- The detection rule is
1 - Problem statement

- The transmitted constellation, $\text{Bs}$, and the received one, $\text{HBs}$, are related by the linear transformation of the channel.

- With ML detection, the performance is dominated by the worst pair-wise error probability:

$$p_{\text{ep}}(n,m) = \Pr(\mathcal{L}(s_m) > \mathcal{L}(s_n)|s_n) = Q\left(\sqrt{\frac{d^2_{n,m}}{2\sigma^2}}\right)$$

where $d^2_{n,m}$ is the distance between two received constellation points, $d^2_{n,m} = \|\text{HB}s_n - \text{HB}s_m\|^2$.
1 - Problem statement

• Our optimization criterion is

$$\arg \min_B \max_q \text{pep}_q = \arg \max_B \min_q d_q^2 = \arg \max_B d_{\min}^2$$

where the squared distance is given by

$$d_{n,m}^2 = \|H B s_n - H B s_m\|^2 = e_{n,m}^H B^H R_H B e_{n,m}$$

with $R_H = H^H H$ and $e_{n,m} = s_n - s_m$

• We obtain

$$\max_B \min_e e^H B^H R_H B e, \quad \text{s.t.} \quad e \in \mathcal{E}, \quad \text{Tr} BB^H \leq P_T$$
1 - Problem solution

- The optimal transmitter matrix is \( \mathbf{B}^* = \mathbf{U}_H \Lambda_Q^{1/2} \mathbf{V}^H \)
1 - Problem solution

- The optimal transmitter matrix is

\[ B^* = U_H \Lambda_Q^{1/2} V^H \]

- The left eigenvectors are the same as in the capacity achieving solution
  - Transmission along the eigenmodes of the channel
1 - Problem solution

- The optimal transmitter matrix is

$$B^* = U_H \Lambda_Q^{1/2} V^H$$

- The power allocation is completely different

$$\lambda_{Q,j}^* = \frac{t^* \beta_j}{\lambda_{H,j}}$$

- Similar to the ZF solution (inversion of the gains)
- The parameter $t$ is calculated to satisfy the power constraint
- The main difference is the presence of the beta factors
  - Can be equal to zero $\rightarrow$ Some eigenmodes are disabled
  - For a given active set, they are constant values, which depend on the geometry of the constellation
  - The beta factors define the aspect ratio of the received constellations
1 - Problem solution

- The optimal transmitter matrix is
  \[ B^* = U_H \Lambda_Q^{1/2} V_H \]
- The power allocation is completely different

\[ \lambda_{Q,j}^* = \frac{t^* \beta_j}{\lambda_{H,j}} \]
\[ \lambda_{Q,j}^* \lambda_{H,j} = t^* \beta_j \]
\[ \lambda_{Q,j}^* = \left( \mu - \frac{1}{\lambda_{H,j}} \right)^+ \]

- Similar to the ZF solution (inversion of the gains)
- The parameter \( t \) is calculated to satisfy the power constraint
- The main difference is the presence of the \( \beta \) factors
  - Can be equal to zero \( \Rightarrow \) Some eigenmodes are disabled
  - For a given active set, they are constant values, which depend on the geometry of the constellation
  - The \( \beta \) factors define the aspect ratio of the received constellations
1 - Problem solution

- The optimal transmitter matrix is

\[ B^* = U_H \Lambda_Q^{1/2} V^H \]

- The right eigenvectors matrix \( V \) is very difficult to calculate (in general, exhaustive search is needed). The purpose of this matrix is to linearly transform the constellation points in \( s \) into a new constellation \( Vs \)
1 - Problem solution for a simple case

- Two QPSK data streams are to be sent (2 eigenmodes)
- The optimal solution has a closed form expression

- One active stream
  \[ \frac{\lambda_{H,2}}{\lambda_{H,1}} < 0.097 \]
  \[ \beta_1 = 1 \quad \beta_2 = 0 \]

- Two active streams
  \[ \frac{\lambda_{H,2}}{\lambda_{H,1}} > 0.097 \]
  \[ \beta_1 = 1 \]
  \[ \beta_2 = 3 - 2\sqrt{2} \]
1 - Performance evaluation

- Comparison for 4 bit per channel use (2QPSK streams)
1 - Performance evaluation

- Comparison for 4 bit per channel use (2QPSK streams)

![Performance Comparison of different transmission schemes with ML detection](image)
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• Conclusions
2 - Capacity results

• Single user MIMO capacity with incomplete CSI has been studied for the cases of statistical knowledge where the CSI consisted of:
  - Channel covariance and/or mean knowledge

• Motivated by system design with power feedback, we consider a different kind of incomplete CSI, which in this case is not statistical but instantaneous:
  - Magnitude knowledge
  - Phase uncertainty

\[
[H]_{ij} = m_{ij} e^{i\theta_{ij}}
\]

\[
H = M \odot P
\]
2 - Uncertainty model application

• Dynamic scenarios (Ergodic formulation)
  – Scenario with direct line of sight, where the mobile user is moving slowly and there exists moderate to high uncorrelation of the channel phases as described in [Paulraj,Kailath,88]
  – While the magnitude matrix $\mathbf{M}$ would remain approximately constant, the phases in $\mathbf{P}$ would change very rapidly because of the relative movement

• Static scenarios (Compound formulation)
  – Scenario where no significant channel variability may occur during the transmission of the message and where the transmitter is only able to estimate accurately the magnitude matrix $\mathbf{M}$. For example, due to different track lengths in the RF front ends at each antenna
2 - Capacity results

- Instantaneous mutual information
  \[ \Psi(Q, M, P) = \log \det \left( I + \sigma^{-2}(M \otimes P)Q(M \otimes P)^H \right) \]

- Ergodic formulation
  - Mutual information
    \[ I_E(Q, M) = \mathbb{E}_P \Psi(Q, M, P) \]
  - Capacity
    \[ C_E = \sup_Q I_E(Q, M) \]

- Compound formulation
  - Mutual information
    \[ I_C(Q, M) = \inf_{P \in \mathcal{P}} \Psi(Q, M, P) \]
  - Capacity
    \[ C_C = \sup_Q I_C(Q, M) \]
2 - Capacity results

• For both cases (ergodic and compound) the optimal covariance matrix is diagonal (power allocation)

\[ Q^* = \Lambda_Q \]

• Since no phase information is available, the eigenmodes of the channel are not well defined
• It seems reasonable not to choose any privileged direction

• Formally, the optimality of the diagonal structure is proven using the invariance properties of the mutual information functions
2 - Ergodic formulation

- Closed form solution exists for the 2x2 case
- For the general case the solution has to be calculated numerically

\[ I_E(\Lambda_Q, M) = \mathbb{E}_P \Psi(\Lambda_Q, M, P) \]
\[ C_E = \sup_{\Lambda_Q} I_E(\Lambda_Q, M) \]

- The objective function is difficult to be evaluated
  - Finite sample solution (Set of phases \( \mathcal{M} \subset \mathcal{P} \))
    \[ I_E(\Lambda_Q, M) \approx I_E^{\text{num}}(\Lambda_Q, M) = \frac{1}{|\mathcal{M}|} \sum_{P \in \mathcal{M}} \Psi(\Lambda_Q, M, P) \]
  - Results from RMT
    \[ \lim_{n_T \to \infty, n_R \to \infty} I_E(\Lambda_Q, M) - \bar{I}_E(\Lambda_Q, M) = 0, \ a.s. \]
2 - Ergodic approximation with RMT

- Basic development (follows the procedure of W. Hachem)

\[ I_E(\Lambda_Q, M) = \mathbb{E}_P \log \det \left( I + \sigma^{-2} (M \circ P) \Lambda_Q (M \circ P)^H \right) \]

\[ I_E(\Lambda_Q, M) = \mathbb{E}_P \sum_{i=1}^{n_R} \log \left( 1 + \sigma^{-2} \zeta_i \right) \]

\[ I_E(\Lambda_Q, M) = \int_{\sigma^2}^{\infty} \left( \frac{1}{\xi} - \mathbb{E}_P m_{\mu}(-\xi) \right) d\xi \]

\[ I_E(\Lambda_Q, M) \simeq \bar{I}_E(\Lambda_Q, M) = \int_{\sigma^2}^{\infty} \left( \frac{1}{\xi} - m_{\nu}(-\xi) \right) d\xi \]
2 - Ergodic approximation with RMT

- Basic development (follows the procedure of W. Hachem)

\[ I_E(\Lambda_Q, M) = \mathbb{E}_P \log \det \left( I + \sigma^{-2}(M \otimes P)\Lambda_Q(M \otimes P)^H \right) \]

\[ I_E(\Lambda_Q, M) = \mathbb{E}_P \sum_{i=1}^{n_R} \log \left( 1 + \sigma^{-2} \zeta_i \right) \]

**Deterministic approximation of the Stieltjes transform**

\[ I_E(\Lambda_Q, M) = \int_{\sigma^2}^{\infty} \left( \frac{1}{\xi} - \mathbb{E}_P m_\mu(-\xi) \right) d\xi \]

\[ I_E(\Lambda_Q, M) \approx \tilde{I}_E(\Lambda_Q, M) = \int_{\sigma^2}^{\infty} \left( \frac{1}{\xi} - m_\nu(-\xi) \right) d\xi \]
2 - Ergodic approximation with RMT

- Convergence of the **random** and **deterministic** pdfs
2 - Ergodic approximation with RMT

- Optimization of

$$C_E = \sup_{\Lambda_Q} \bar{I}_E(\Lambda_Q, M) = \int_{\sigma^2}^{\infty} \left( \frac{1}{\xi} - m_\nu(-\xi) \right) d\xi$$

The optimization is done in the **deterministic** approximation.

As a result, the random pdf is also optimized.
2 - Numerical examples (ergodic)

- Evaluation of the ergodic capacity

![Numerical Evaluation of the Ergodic Capacity](image)

- 8x8
- 4x4

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PhD dissertation
2 - Numerical examples (ergodic)

- Value of magnitude knowledge in the ergodic capacity
2 - Compound formulation

- Closed form solution exists for the $2 \times n_R$ case
- For the general case the solution has to be calculated numerically

$$I_C(\Lambda_Q, M) = \inf_{P \in \mathcal{P}} \Psi(\Lambda_Q, M, P) \quad C_C = \sup_{\Lambda_Q} I_C(\Lambda_Q, M)$$

- The objective function is difficult to be evaluated
  - Finite sample solution (Set of phases $\mathcal{M} \subset \mathcal{P}$)
    $$I_C(\Lambda_Q, M) \approx I_C^{\text{num}}(\Lambda_Q, M) = \inf_{P \in \mathcal{M}} \Psi(\Lambda_Q, M, P)$$
  - Taylor approximation
    $$I_C(\Lambda_Q, M) \approx \log \det (I + \sigma^{-2} M \Lambda_Q M^H)$$
2 - Numerical examples (compound)

• Evaluation of the compound capacity

![Numerical Evaluation of the Compound Capacity](chart.png)
2 - Numerical examples (compound)

- Value of magnitude knowledge in the compound capacity
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  - Perfect CSI and single user communications
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- Conclusions
3 - Imperfect CSI

- We study the effects of imperfect CSI on the achievable rates of the Tomlinson-Harashima Precoder (THP)

- The THP is chosen because it is a versatile structure which allows us to deal with the single and multiuser scenarios simultaneously
3 - Tomlinson-Harashima precoder

- For the single user scenario case, it becomes

**Contribution:**
Allowing different values for the entries of $t$

$t = [t_1 \ldots t_N]$

$P^c_k = 2t^2_k/3$

$r = Mt [s + GFn]$

$r_{\hat{B}} = Mt [s + Lx_c + G\tilde{n}]$

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3 - Tomlinson-Harashima precoder

- For the multiuser scenario case, it becomes

\[ \hat{H} = H + \Delta \]

\[ \hat{B} = f(\hat{H}) = B + \Delta_B \]

\[ \hat{F} = g(\hat{H}) = F + \Delta_F \]

\[ r = M_t [s + GFn] \]

\[ r_{B,F} = M_t [s + \xi(s) + Gn] \]
3 - Achievable rates problem

• The problem is to design the modulo operation $t$ to optimize the achievable rates (in the multiuser case we optimize the sum rate)

$$C_{THP}^{rob} = \max_t \min_{\Delta} \sum_{k=1}^{N} I(s_k; r_k)$$

s.t. $$\sum_{k=1}^{N} \frac{2t_k^2}{3} \leq P_T$$

$$\Delta \in \mathcal{R}$$

• The solution has to be calculated numerically
• Introducing some approximations the solution can be found in closed form
3 - Solution in the single user case

- Mutual information loss comparison

\[ t_k = \sqrt{\frac{3P_T}{2|G|}} \]

or

\[ t_k = 0 \]
3 - Solution in the multiuser case

- The (approximate) solution is user selection and uniform modulo operation.

\[
 t_k = \sqrt{\frac{3P_T}{2|G|}} \\
\text{or} \\
 t_k = 0
\]
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• Conclusions
4 - Capacity results

- The capacity region for the multiuser (broadcast) linear MIMO channel with Gaussian noise has been recently found [Weingarten, Steinberg, Shamai 04-06].

- For the imperfect CSI case almost no results exist

- We focus on practical transmission schemes
  - The downlink problem
4 - The downlink problem

• Design a linear transmitter for the broadcast channel such that:

  - Each user has a QoS constraint
  - The objective is to transmit the minimum power

\[
\begin{align*}
\text{minimize} & \quad P_T \\
\text{subject to} & \quad \text{qos}_i \geq \text{qos}_i^0, \quad \forall i \in \{1, 2, \ldots, n_U\}.
\end{align*}
\]
4 - Communications set-up

\[ H = [h_1, \ldots, h_{n_U}]^H \]

\[ y_i = [y]_i = h_i^H x + n_i, \]
4 - The solution

• The downlink problem has been studied and solved by two research groups (Bengtsson and Boche)

• Both groups considered that:
  – The QoS indicator is the SINR
  – Perfect CSI is available at the transmitter and receiver

• For the imperfect CSI case, this problem has been addressed and solved by two research groups (Gershman and Bengtsson)
  – The uncertainty is in the correlation matrix

\[ \tilde{R}_H = R_H + \Delta \]
4 - Our problem

• We want to solve the downlink problem for imperfect CSI in the channel estimate

\[ H = \tilde{H} + \Delta \]

• We assume that the actual channel belongs to an uncertainty region around the channel estimate

\[ \Delta \in \mathcal{R} \]
4 - Problem statement

- The problem becomes

\[
\begin{align*}
\text{minimize} & \quad P_T \\
\text{subject to} & \quad \text{qos}_i \geq \text{qos}_i^0, \quad \forall i \in \{1, 2, \ldots, n_U\}, \\
& \quad \forall \Delta \in \mathcal{R}.
\end{align*}
\]

- To be able to find a solution we have to simplify the architecture:
  - The linear transmitter is divided in two parts and only one of them is a design parameter
    \[
    T = BP^{1/2}
    \]
  - The receivers estimate the symbol by a simple division
4 - Simplification at the transmitter

- Transmitter architecture

\[
P_T(P) = \text{Tr } BPB^H
\]

\[
B_{ZF} = \tilde{H}^H \left( \tilde{H} \tilde{H}^H \right)^{-1}
\]

\[
B_{WF} = \left( \tilde{H}^H \tilde{H} + \alpha I \right)^{-1} \tilde{H}
\]

\[
B = I
\]
4 - Receivers simplification

- Receiver architecture

\[ y_i \xrightarrow{\text{\(\tilde{h}_i^H b_i \sqrt{p_i}\)}}^{-1} S_i = s_i + \frac{(\tilde{h}_i^H \tilde{B}_i + \delta_i^H B)P^{1/2} s + n_i}{\tilde{h}_i^H b_i \sqrt{p_i}} \]

- The QoS indicator is chosen to be the inverse of the MSE (effective SINR)

\[ \text{mse}_i = \mathbb{E}|s_i - \hat{s}_i|^2 \]

\[ \frac{1}{\text{mse}_i} \triangleq \text{esinr}_i. \]
4 - Formal problem statement

- The problem can then be expressed as:

$$\text{minimize}_{P} \quad P_T = \text{Tr}BPB^H$$

subject to

$$\text{esinr}_i(\Delta, P \geq \text{esinr}^0_i, \forall i \in \{1, 2, \ldots, n_U\},$$

$$\forall \Delta \in \mathcal{R}.$$ 

- We managed to reformulate the problem in convex form no matter what is the shape of the uncertainty region.

- To go further we need to define:
  - The linear transmitter matrix: $B_{ZF}$
  - The shape and size of the uncertainty region
4 - The uncertainty region

\[ H = \tilde{H} + \Delta \]

- The shape and size of the uncertainty region must be connected with the physical phenomenon producing the error
  - Estimation Gaussian noise
  - Quantization effects
  - Combinations of both
4 - Estimation Gaussian noise

- Each user estimates its own channel and feeds the estimation back to the transmitter (assumed error free)
- The estimate is a version of the actual channel corrupted with AWGN
- The uncertainty region is a set of spherical regions, centered at the estimate of the channel of each user

\[ \cdot \hat{h}_1 \quad R_2 \quad \cdot \hat{h}_2 \quad \cdot \hat{h}_3 \]

- Since the error is unbounded there is a certain probability that the actual channel is outside the uncertainty region (outage event declared)
4 - Estimation Gaussian noise

• For this particular case a closed form solution exists (derived from the KKT)

\[ p^*_i = \text{esinr}_i^0 \sigma^2 + \text{esinr}_i^0 \mu R_i^2 = \text{esinr}_i^0 (R_i^2 \mu + \sigma^2) \]

• The \( \mu \) parameter is the solution to a fixed point equation (can be calculated numerically in a very efficient way)

• The power allocation with perfect CSI is given by

\[ p^*_{i, \text{perfect}} = \text{esinr}_i^0 \sigma^2 \]

• The price of the robust design is \( \text{esinr}_i^0 \mu R_i^2 \)
4 - Quantization effects

- Each user quantizes the actual realization of the channel and feeds it back to the transmitter (possibly through a digital link)

\[ \tilde{h}_i = Q\{h_i\} \]
4 - Quantization effects

- In this case the convex optimization problem can be simplified to a linear program
  - Linear objective
  - Linear constraints
  - In general, no closed form is available

- There are powerful tools to solve this kind of problems

\[
\begin{align*}
\text{minimize} \quad & \quad \operatorname{Tr} B_{ZF} P B_{ZF}^H, \\
\text{subject to} \quad & \quad v_{m,i}^H B_{ZF} P B_{ZF}^H v_{m,i} - \text{mse}_i^0 p_i + \sigma^2 \leq 0, \quad \forall i
\end{align*}
\]
4 - Combination of regions

- In the most general setup each user
  1. Estimates the channel
  2. Quantizes this estimate

- In this case, the uncertainty region becomes more involved
4 - Combination of regions

• The problem is still convex, but it doesn’t have a particular structure

\[
\begin{align*}
\text{minimize} & \quad \operatorname{Tr} \mathbf{B}_{ZF} \mathbf{P} \mathbf{B}_{ZF}^H, \\
\text{subject to} & \quad (s_{m,i}^*(\mathbf{P}) + \mathbf{v}_{m,i})^H \mathbf{B}_{ZF} \mathbf{P} \mathbf{B}_{ZF}^H (s_{m,i}^*(\mathbf{P}) + \mathbf{v}_{m,i}) - \text{mse}_i^0 p_i + \sigma^2 \leq 0,
\end{align*}
\]

• Numerical methods are applied to compute the solution

• The computational load is in the calculation of the restriction
4 - Feasibility test

• The problem

\[
\begin{align*}
\minimize_{\mathbf{P}} \quad & P_T = \text{Tr} \mathbf{B} \mathbf{P} \mathbf{B}^H \\
\text{subject to} \quad & \text{esinr}_i(\Delta, \mathbf{P}) \geq \text{esinr}_i^0, \quad \forall i \in \{1, 2, \ldots, n_U\}, \\
& \forall \Delta \in \mathcal{R}.
\end{align*}
\]

may become infeasible if there exists no \( \mathbf{P} \) matrix such that the constraints can be simultaneously fulfilled.

• We derived a feasibility check in the form of a convex problem, which should be solved previously.
4 - Problem solution algorithm

Perform feasibility test

Feasible?

YES

Solve the problem

NO

Relax the constraints or declare an outage
4 - Numerical examples

- Feasibility region

2 user scenario
4 - Numerical examples

- Comparison for different power allocation policies

\[ B = B_{ZF} \]

8 user scenario
4 - Numerical examples

- Comparison for different choice of transmitter matrix $B$

8 user scenario

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February 14th, 2007
Outline

• Communications set-up

• Cases of study
  – Perfect CSI and single user communications
  – Incomplete CSI and single user communications
  – Imperfect CSI, from single to multiuser communications
    – Imperfect CSI and multiuser communications

• Conclusions
Outline

• Communications set-up

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• Conclusions
Conclusions

• The impact of the CSI on the design and analysis of multi-antenna systems has been characterized

• Perfect CSI (single user)
  – Design of linear transmitter for practical constellations

• Incomplete CSI (single user)
  – Analysis of the performance limits for the case of magnitude knowledge and phase uncertainty

• Imperfect CSI (single and multiuser)
  – Analysis of the achievable rates of the Tomlinson-Harashima precoder in single and multiuser scenarios

• Imperfect CSI (multiuser)
  – Design of a robust linear transmitter to minimize the transmitted power while guaranteeing QoS to the users
Thank you very much!