ROBUST POWER ALLOCATION TECHNIQUES FOR MIMO SYSTEMS UNDER MODULUS CHANNEL KNOWLEDGE AT THE TRANSMITTER

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ABSTRACT

In this paper, we study a family of robust power allocation techniques for multi-input multi-output communication systems with incomplete channel state information at the transmitter side. Precisely, we focus our attention to the case where the transmitter has access only to the modulus of the channel coefficients. We present the robust techniques under two main approaches: the bayesian (or stochastic) point of view, and the maximin (or worst-case) criterion. The formulation developed in this paper holds for flat fading channels, and also for frequency selective channels; in this latter case, through a multicarrier perspective.

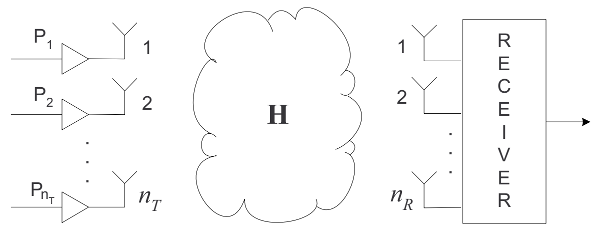

Fig. 1. Optimum transmit architecture for a MIMO system with phase uncertainty at the transmitter

1. INTRODUCTION

The design of multi-input multi-output (MIMO) communication systems depends not only on the chosen figure of merit (MSE, SINR, BER, or capacity), but also on the quantity and the quality of the channel state information (CSI) that is made available at the transmitter side. As far as capacity is concerned, in his popular work [1], Telatar stated that, when the transmitter has no CSI, uniform power allocation is the most reasonable approach to achieve capacity. The optimality of the uniform power allocation scheme was later proven in [2] using game-theoretic justifications. For the case where the transmitter has perfect CSI, it is also well known, [1], that the optimum transmit strategy consists in splitting the signal among a set of beamformers giving each one a fraction of transmission power according to a "waterfilling" algorithm.

The capacity of multiantenna systems with partial CSI at the transmitter side has been studied, for example, in [3],[4],[5], in which a multi-input single-output (MISO) architecture is considered, and it is assumed that the transmitter has access only to either the mean value or the covariance of the channel vector. In [6], the effect of partial side information at the transmitter is analyzed for a generic MIMO case. One of the most important conclusions of that paper is that capacity can be achieved by a structure that first maps source symbols into space-time codewords independently from the CSI, and then weights these codewords as a function of the CSI at the transmitter.

In [7], a different kind of partial CSI at the transmitter side is considered, since it is assumed that the transmitter has perfect knowledge of the modulus of the complex channel coefficients but a complete lack of knowledge of the channel phases. Many interesting practical situations fit in this model. In time division duplex (TDD) mobile communication systems, for example, if electromagnetic reciprocity between uplink and downlink channels is exploited, then perfect knowledge of the uplink channel yields perfect knowledge of the downlink response. In practice, however, the channel reciprocity is broken because uplink and downlink channels are seen through different radiofrequency (RF) front-ends. To restore reciprocity, calibration architectures that compensate for the amplitude and phase response of each RF front-end must be used. Although calibration of the amplitude response is relatively easy, phase response calibration becomes substantially more complicated (due, for example, to different lengths of the tracks between the oscillator and the different mixers). Hence, in such architectures only perfect knowledge of the channel amplitude can be reasonably assumed. For that particular case of incomplete CSI, the optimal transmission architecture in terms of mutual information is to perform independent signaling through the antennas, *i.e.* a simple power allocation scheme (see Fig. 1). This result will be the starting point of the present paper. Here, we will try to optimize the mutual information between the transmitter and the receiver by performing transmission power allocation. The worst-case and the bayesian viewpoints [8] will be the two approaches that will be used to cope with the problem of having incomplete CSI, thus giving rise to robust designs. On one hand, the maximin (or worst-case) design [9], [10] guarantees a certain system performance for any possible channel compatible with the incomplete CSI, [11]. On the other hand, the bayesian design, as in [3] or [4], guarantees a certain system performance averaged over the statistics of the unknown part of the incomplete CSI.

The remainder of the paper is organized as follows. In Section 2 the system model is presented. The setup of the problems for the bayesian and maximin approaches is described in Sections 3 and 4, respectively. In section 5, the problem statement is extended to frequency selective MIMO channels through a multicarrier approach. Finally, in Section 6 some simulations results are presented, and in Section 7 the conclusions are given.

2. SYSTEM MODEL

Initially, we consider a narrowband multiplexing system with n_T transmit and n_R receive antennas corrupted with additive white Gaussian noise. Let us define $\mathbf{x} \in \mathbb{C}^{n_T}$ as the transmit signal vector, $\tilde{\mathbf{H}} \in \mathbb{C}^{n_R \times n_T}$, as the channel matrix, and $\mathbf{n} \in \mathbb{C}^{n_R}$, as the noise vector, modeled as a circularly symmetric Gaussian distributed random vector with zero mean and covariance matrix $\mathbb{E}\left\{\mathbf{n}\mathbf{n}^H\right\} = \sigma^2\mathbf{I}$, which is supposed known at the transmitter.

As it was stated in the introduction, we assume that the transmitter only has access to the channel coefficients, $\tilde{h}_{kl} = m_{kl}e^{j\tilde{\theta}_{kl}}$, through an unknown phase shift, ϕ_{kl}, which depends on the difference of the phase responses between the uplink and downlink RF front-ends, defining the *effective* channel response, h_{kl},

$$h_{kl} = \tilde{h}_{kl}e^{j\phi_{kl}} = m_{kl}e^{j(\phi_{kl}+\tilde{\theta}_{kl})} = m_{kl}e^{j\theta_{kl}}. \qquad (1)$$

Instead of trying to calibrate the RF phase responses, we assume that θ_{kl} is unknown at the transmitter side. To separate the known from the unknown part of the elements of $\mathbf{H}$, $[\mathbf{H}]_{kl} = h_{kl}$, we define

$$\mathbf{H}(\boldsymbol{\Theta}) \equiv \mathbf{M} \odot \mathbf{P}(\boldsymbol{\Theta}),$$

where $[\mathbf{P}(\boldsymbol{\Theta})]_{kl} = e^{\mathrm{j}[\boldsymbol{\Theta}]_{kl}}$ and $\odot$ denotes the Hadamard element-wise matrix product. The unknown channel phases, θ_{kl}, are modeled as i.i.d. uniform random variables in the interval $[-\pi, \pi)$ as suggested in, *e.g*, [12], as the unknown, but fixed, phase shift, ϕ_{kl}, does not change the uniform distribution of $\tilde{\theta}_{kl}$.

The received signal, $\mathbf{y} \in \mathbb{C}^{n_R}$, for this model can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = [\mathbf{h}_1\mathbf{h}_2 \cdots \mathbf{h}_{n_T}]\,\mathbf{x} + \mathbf{n}, \qquad (2)$$

where we assume that $\mathbf{H}$ is perfectly known at the receiver.

2.1. Frequency selective MIMO channel

In this section we will consider that the MIMO channel is frequency selective. As it is well known, a frequency selective MIMO channel can be dealt with by taking a multicarrier approach, which is a capacity lossless structure and each subcarrier can be treated as a flat fading MIMO channel. We define $\tilde{h}^t_{pq}[n]$, $0 \le n \le N_0 - 1$, as the discrete time response between q-th transmit and p-th receive antennas. Thus, the frequency response between the (q,p) pair in k-th subcarrier, $\tilde{h}^f_{pq}[k]$, will be

$$\tilde{h}^f_{pq}[k] = \sum_{n=0}^{N_0-1} \tilde{h}^t_{pq}[n]e^{-j2\pi\frac{kn}{K}}, \quad 0 \le k \le K-1, \qquad (3)$$

where K is the number of total subcarriers. Since there is an unknown phase difference between the real channel response, $\tilde{h}^t_{pq}[n]$, and the channel available at the transmitter, $h^t_{pq}[n]$, the response at k-th subcarrier seen by the transmitter will be

$$\begin{aligned} h^t_{pq}[n] = \tilde{h}^t_{pq}[n]e^{j\phi_{pq}} \rightarrow \\ \rightarrow h^f_{pq}[k] = \tilde{h}^f_{pq}[k]e^{j\phi_{pq}} = \\ = m^f_{pq}[k]e^{j(\phi_{pq}+\tilde{\theta}^f_{pq}[k])} = m^f_{pq}[k]e^{j\theta_{pq}[k]}, \end{aligned} \qquad (4)$$

which implies that the uncertainty in the channel phases in the flat fading case can be directly applied to the multicarrier case leading to the same model, but in a per-carrier basis. If we define $[\mathbf{M}_k]_{pq} = |h^f_{pq}[k]| = m^f_{pq}[k]$ then the MIMO frequency response matrix at k-th subcarrier will be

$$\mathbf{H}_k = \mathbf{M}_k \odot \mathbf{P}(\boldsymbol{\Theta}_k), \qquad (5)$$

where $\boldsymbol{\Theta}_k$ is assumed to be unknown to the transmitter.

2.2. Figure of merit

In this work we are interested in maximizing the mutual information between the transmitted and the received signals. For a particular realization of the flat-fading channel matrix, $\mathbf{H}$, the instantaneous mutual information, assuming that we are performing power allocation (which is a capacity lossless structure for phase uncertainty at the transmitter) as depicted in Fig. 1, is

$$I\left(\boldsymbol{\Lambda}, \mathbf{H}\right) = \log\det\left(\mathbf{I} + \mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^H\right), \qquad (6)$$

where $\mathbf{H} = \mathbf{M} \odot \mathbf{P}\left(\boldsymbol{\Theta}\right)$ and $\boldsymbol{\Lambda}$ is a positive semidefinite diagonal matrix.

Thanks to the orthogonality between the subcarriers, the instantaneous mutual information in the multicarrier case can be easily found as

$$I\left(\{\boldsymbol{\Lambda}_k\}, \{\mathbf{H}_k\}\right) = \sum_k \log\det\left(\mathbf{I} + \mathbf{H}_k\boldsymbol{\Lambda}_k\mathbf{H}_k^H\right), \qquad (7)$$

with $\mathbf{H}_k = \mathbf{M}_k \odot \mathbf{P}\left(\boldsymbol{\Theta}_k\right)$ and $\boldsymbol{\Lambda}_k$ is a positive semidefinite diagonal matrix.

3. BAYESIAN ROBUST APPROACH

In the bayesian robust approach we are interested in finding the power allocation that maximizes the figure of merit averaged over the statistics of the unknown part of the incomplete CSI and subject to a power constraint. The problem can be formally stated as

$$\begin{aligned} R^B(\gamma, \mathbf{M}) \triangleq \sup_{\boldsymbol{\Lambda}} \quad & \mathbb{E}_{\boldsymbol{\Theta}} I\left(\boldsymbol{\Lambda}, \mathbf{M} \odot \mathbf{P}\left(\boldsymbol{\Theta}\right)\right) \\ \text{s.t.} \quad & \mathrm{Tr}\left(\boldsymbol{\Lambda}\right) = \gamma, \\ & \lambda_i \ge 0, \qquad 1 \le i \le n_T, \end{aligned} \qquad (8)$$

where $\lambda_i = [\boldsymbol{\Lambda}]_{ii}$. The maximization is over the set of diagonal positive semidefinite matrices with $\gamma \equiv P_T/\sigma^2$, where P_T is the transmitted power and σ^2 is the noise variance. The expectation is taken over the distribution of the phases of the entries of $\mathbf{H}$.

In [7], this convex optimization problem was solved, in a closed form, for the 2×2 MIMO case. The analytical evaluation of the expectation gets very involved as the number of transmitters or receivers gets bigger than 2. In addition, the numerical evaluation of the expectation is computationally hard, since it involves integrating over multiple-dimensions.

4. MAXIMIN ROBUST APPROACH

In the maximin robust approach we are interested in finding the power allocation, subject to a total power constraint, that maximizes the worst-case, in terms of channel phases, of the figure of merit. This problem can be formally stated as

$$\begin{aligned} R^M(\gamma, \mathbf{M}) \triangleq \sup_{\boldsymbol{\Lambda}} \quad & \inf_{\boldsymbol{\Theta} \in \mathcal{T}} I\left(\boldsymbol{\Lambda}, \mathbf{M} \odot \mathbf{P}\left(\boldsymbol{\Theta}\right)\right) \\ \text{s.t.} \quad & \mathrm{Tr}\left(\boldsymbol{\Lambda}\right) = \gamma, \\ & \lambda_i \ge 0, \qquad 1 \le i \le n_T, \end{aligned} \qquad (9)$$

where $\mathcal{T} \equiv \{\mathbf{\Theta} \mid \theta_{ij} \in [-\pi, \pi)\}$. Notice that if (9) is written as a minimization in standard form [8] the resulting problem is convex. Before we proceed with the analysis of this case, we need to introduce some definitions that will prove to be very useful in the sequel.

4.1. Introduction of function $\mathcal{D}$

First of all we define

$$\mathcal{D}(\mathbf{\Lambda}, \mathbf{H}) = 1 + \sum_i \lambda_i \alpha_{ii} + \sum_{i,j>i} \lambda_i \lambda_j \left(\alpha_{ii}\alpha_{jj} - |\alpha_{ij}|^2\right), \quad (10)$$

where $\alpha_{ij} = \mathbf{h}_i^H \mathbf{h}_j$. Notice that $|\alpha_{ij}|^2$ can be upper bounded as

$$\begin{aligned} |\alpha_{ij}|^2 &= |\mathbf{h}_i^H \mathbf{h}_j|^2 = \left(\sum_k h_{ki}^* h_{kj}\right)\left(\sum_l h_{li} h_{lj}^*\right) = \\ &= \sum_k |h_{ki}|^2 |h_{kj}|^2 + \sum_{k,l\neq k} h_{ki}^* h_{kj} h_{li} h_{lj}^* = \sum_k m_{ki}^2 m_{kj}^2 + \\ &+ \sum_{k,l>k} 2 m_{ki} m_{kj} m_{li} m_{lj} \cos(\theta_{ki} - \theta_{kj} - \theta_{li} + \theta_{lj}) \leq \\ &\leq \sum_k m_{ki}^2 m_{kj}^2 + \sum_{k,l>k} 2 m_{ki} m_{kj} m_{li} m_{lj} = |\mathbf{m}_i^T \mathbf{m}_j|^2. \quad (11) \end{aligned}$$

Noticing that $\alpha_{ii} = \|\mathbf{h}_i\|^2 = \|\mathbf{m}_i\|^2$, and using the upper bound above it can be easily seen that

$$\mathcal{D}(\mathbf{\Lambda}, \mathbf{H}) \geq \mathcal{D}(\mathbf{\Lambda}, \mathbf{M}). \quad (12)$$

Using $\mathcal{D}$ function, the second order Taylor series of the determinant in (6) can be expressed as

$$\det\left(\mathbf{I} + \mathbf{H}\mathbf{\Lambda}\mathbf{H}^H\right) = \mathcal{D}(\mathbf{\Lambda}, \mathbf{H}) + o\left(\sum_{i,j} \lambda_i \lambda_j\right). \quad (13)$$

For the particular case where $\min(n_T, n_R) = 2$ the approximation becomes exact, *i.e.* $\det\left(\mathbf{I} + \mathbf{H}\mathbf{\Lambda}\mathbf{H}^H\right) = \mathcal{D}(\mathbf{\Lambda}, \mathbf{H})$.

4.2. Solution for $n_T = 2$

From what was stated in last section, for the particular case where $n_T = 2$, the minimization in (9) simplifies to

$$\inf_{\mathbf{\Theta}\in\mathcal{T}} I(\mathbf{\Lambda}, \mathbf{H}) = \inf_{\mathbf{\Theta}\in\mathcal{T}} \log \mathcal{D}(\mathbf{\Lambda}, \mathbf{H}) = \log \mathcal{D}(\mathbf{\Lambda}, \mathbf{M}). \quad (14)$$

Thus, the problem in (9) can be reformulated as

$$\begin{aligned} R^M(\gamma, \mathbf{M}) = \sup_{\lambda_1, \lambda_2} \quad & \log \mathcal{D}(\mathbf{\Lambda}, \mathbf{M}) \\ \text{s.t.} \quad & \lambda_1 + \lambda_2 = \gamma, \quad (15) \\ & \lambda_i \geq 0, \qquad 1 \leq i \leq 2, \end{aligned}$$

and, using the KKT conditions and supposing $\|\mathbf{m}_1\|^2 < \|\mathbf{m}_2\|^2$ and $\mathbf{m}_1 \neq k\mathbf{m}_2$, the optimal solution can be readily obtained as

$$\begin{aligned} \lambda_1^\star &= \left(\frac{\gamma}{2} - \frac{\|\mathbf{m}_2\|^2 - \|\mathbf{m}_1\|^2}{2\left(\|\mathbf{m}_1\|^2\|\mathbf{m}_2\|^2 - |\mathbf{m}_2^T\mathbf{m}_1|^2\right)}\right)^+, \\ \lambda_2^\star &= \gamma - \lambda_1^\star \end{aligned}$$

where $(x)^+ = \max(x, 0)$, and where $(\cdot)^\star$ denotes optimal value. The solution to the case where $\|\mathbf{m}_1\|^2 > \|\mathbf{m}_2\|^2$ is symmetric to the solution presented above, and if $\mathbf{m}_1 = k\mathbf{m}_2$ all the power must be given to the best channel. Notice that, although the problem setup is different, a similar result was obtained in [13].

4.3. Approximation for the general case

We can use the second order expression for the determinant

$$\det\left(\mathbf{I} + \mathbf{H}\mathbf{\Lambda}\mathbf{H}^H\right) = \mathcal{D}(\mathbf{\Lambda}, \mathbf{H}) + o\left(\sum_{i,j} \lambda_i \lambda_j\right), \quad (16)$$

to state that $\det\left(\mathbf{I} + \mathbf{H}\mathbf{\Lambda}\mathbf{H}^H\right) \gtrsim \det\left(\mathbf{I} + \mathbf{M}\mathbf{\Lambda}\mathbf{M}^H\right)$ and approximate the maximization problem in (9) by

$$\begin{aligned} R^M(\gamma, \mathbf{M}) \approx \sup_{\mathbf{\Lambda}} \quad & \log \det\left(\mathbf{I} + \mathbf{M}\mathbf{\Lambda}\mathbf{M}^H\right) \\ \text{s.t.} \quad & \mathrm{Tr}(\mathbf{\Lambda}) = \gamma, \quad (17) \\ & \lambda_i \geq 0, \qquad 1 \leq i \leq n_T, \end{aligned}$$

which can be solved numerically in a very efficient way using interior point methods (see further [8]). In the numerical maximization the logarithm function can be eliminated, and in Appendix A there is an expression for the gradient of the determinant in (17) with respect to the diagonal elements of $\mathbf{\Lambda}$, which can be used to improve the efficiency of the numerical optimization procedure. Note also that, if $n_R = 2$, the approximation becomes exact, similarly to the case where $n_T = 2$, although, in general, no closed form expression can be found.

5. MULTICARRIER EXTENSION

5.1. Bayesian Robust Approach

Although the bayesian robust approach is rather limited to the 2×2 case, most of its potential gains are exploited in the multicarrier case. For the multicarrier case the problem can be stated as follows

$$\begin{aligned} R_{\mathrm{MC}}^B(\gamma_{\mathrm{MC}}, \{\mathbf{M}_k\}) \triangleq \sup_{\{\mathbf{\Lambda}_k\}} \quad & \sum_k \mathbb{E}_{\mathbf{\Theta}_k} I\left(\mathbf{\Lambda}_k, \mathbf{M}_k \odot \mathbf{P}(\mathbf{\Theta}_k)\right) \\ \text{s.t.} \quad & \sum_k \mathrm{Tr}(\mathbf{\Lambda}_k) = \gamma_{\mathrm{MC}}, \quad (18) \\ & [\mathbf{\Lambda}_k]_{ii} \geq 0, \qquad 0 \leq k \leq K-1. \end{aligned}$$

Because all the terms in the summation of the objective function in (18) depend on a diferent set of optimization variables, $\mathbf{\Lambda}_k$, and using the definition in (8) the problem can be reformulated in a more compact form, which in addition simplifies the numerical optimization,

$$\begin{aligned} R_{\mathrm{MC}}^B(\gamma_{\mathrm{MC}}, \{\mathbf{M}_k\}) = \sup_{\{\gamma_i\}} \quad & \sum_k R^B(\gamma_k, \mathbf{M}_k) \\ \text{s.t.} \quad & \sum_k \gamma_k = \gamma_{\mathrm{MC}}, \quad (19) \\ & \gamma_k \geq 0, \qquad 0 \leq k \leq K-1. \end{aligned}$$

Notice that $R^B(\gamma, \mathbf{M})$ has the following properties:

- The function $R^B(\gamma, \mathbf{M})$ is differentiable for $\gamma \geq 0$.
- The function $-R^B(\gamma, \mathbf{M})$ is convex (as desired in optimization problems in standard form [8]) for $\gamma \geq 0$.

See Appendix B for details. The calculation of the derivative of $R^B(\gamma, \mathbf{M})$ can be used to reduce the computational load of the optimization algorithm.

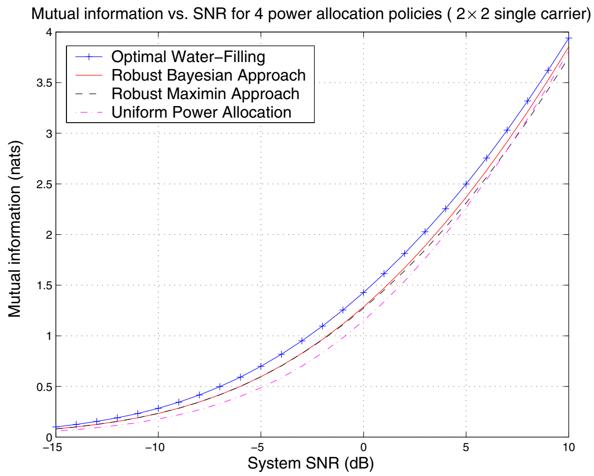

Fig. 2. Achievable mutual information in a 2×2 single carrier MIMO system as a function of mean SNR for different power allocation policies. Notice that in the single carrier case there is no significant differences among them.

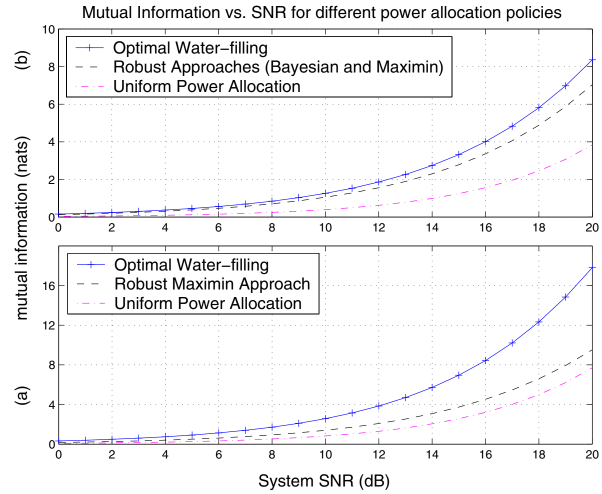

Fig. 3. Mutual information for different MIMO antenna configurations and power allocation policies in a multicarrier system with 48 subcarriers. **(a)** $n_T = 4$ and $n_R = 4$. **(b)** $n_T = 2$ and $n_R = 2$. Notice that, in (b) both robust approaches yield the same mutual information and that the performance of the proposed scheme is closer to the optimum in (b) than in (a).

5.2. Maximin Robust Formulation

The maximin robust problem statement for the multicarrier case is stated in general as

$$\begin{aligned} R^M_{\text{MC}}(\gamma_{\text{MC}}, \{\mathbf{M}_k\}) \triangleq \sup_{\{\boldsymbol{\Lambda}_k\}} \quad & \sum_k \inf_{\boldsymbol{\Theta}_k \in \mathcal{T}} I\left(\boldsymbol{\Lambda}_k, \mathbf{M}_k \odot \mathbf{P}\left(\boldsymbol{\Theta}_k\right)\right) \\ \text{s.t.} \quad & \sum_k \text{Tr}\left(\boldsymbol{\Lambda}_k\right) = \gamma_{\text{MC}}, \qquad (20) \\ & [\boldsymbol{\Lambda}_k]_{ii} \geq 0, \qquad 0 \leq k \leq K-1. \end{aligned}$$

However, in practice the objective function used is approximated, as explained in Section 4.3, by

$$\sum_k \log\det\left(\mathbf{I} + \mathbf{M}_k \boldsymbol{\Lambda}_k \mathbf{M}_k^H\right). \qquad (21)$$

With this simplification the optimization problem in (20) can be solved numerically very efficiently with the use of the before mentioned interior-point methods. In the simulations section it will be shown that, even though the objective function in the maximization is approximated, the resulting power allocation performs very well.

6. SIMULATIONS

Firstly, for the frequency flat channel model, the entries of the channel matrix $\mathbf{H}$ have been assumed to be zero mean unit variance i.i.d. circularly symmetric complex gaussian random variables. In Fig. 2, the obtained mean[1] mutual information is plotted as a function of mean SNR for the bayesian and maximin robust power allocation policies for $n_T = n_R = 2$. In addition, to put into context the obtained results of our approach, the mutual information that can be achieved when perfect and no CSI are available at the transmitter have been also plotted. Although no significant differences among the simulated power allocation policies are observed, as the system SNR grows, the maximin approach yields a lower mean mutual information than the bayesian approach because the former has to guarantee that the mutual information will not be below a certain value for every realization of the channel, where as the latter does not.

Secondly, for the frequency selective channel model, we have chosen the European standard HIPERLAN/2-A for WLAN [14]. It is based on the multicarrier modulation OFDM (a total of 64 carriers, and 48 of them are active). In Fig. 3, the mutual information is plotted as a function of SNR for $n_T = n_R = 2$, (b), and $n_T = n_R = 4$, (a). It can be seen that the differences between the different power allocation policies are accentuated in the multicarrier case. In addition, the gain of the robust approaches with respect to uniform power allocation is higher in Fig. 3(b) than in (a). The authors conjecture that this is due to the fact that the knowledge of only the modulus of the entries of $\mathbf{H}_k$ loses relative importance, with respect to the full knowledge of $\mathbf{H}_k$, as the number of transmit or receive antennas grows. Notice for example that, for $n_T = n_R = 1$, it is equivalent to know $\mathbf{H}_k$ (which is a number) or just $|\mathbf{H}_k|$ because, in SISO-OFDM, the knowledge of the channel power, *i.e.* $|\mathbf{H}_k|^2$, for each subcarrier is enough to achieve capacity.

7. CONCLUSIONS

In this paper we have presented a family of power allocation techniques for a variety of MIMO channels in the form of convex optimization problems. In general, the solution of these problems has to be found through a numerical optimization procedure. We have considered two approaches: the bayesian approach and the maximin approach, both for single and multicarrier MIMO channels. It has been shown that, although in the single carrier case

[1] Averaged over the statistics of the MIMO channel.

the knowledge of the channel coefficients modulus does not give a significant increase in the mutual information with respect to the uniform power allocation policy, in the multicarrier case the gains are considerably larger, and even get close to the maximum of the mutual information which can be achieved only when perfect CSI is available at the transmitter. It is also worth to remark that, in the multicarrier case, for the 2×2 MIMO case the bayesian and the maximin approaches perform extremely closely, as the two plots overlap.

A. GRADIENT OF $\det(\mathbf{I} + \mathbf{M}\mathbf{\Lambda}\mathbf{M}^T)$

We define $\mathbf{X} = \mathbf{I} + \mathbf{M}\mathbf{\Lambda}\mathbf{M}^T$. Using the expression

$$\frac{\partial |\mathbf{X}|}{\partial \mathbf{X}} = |\mathbf{X}|\mathbf{X}^{-T} \tag{22}$$

from [15], and recalling that

$$\frac{\partial [\mathbf{X}]_{ij}}{\partial \lambda_k} = \delta_{ki} \left[\mathbf{M}^T\mathbf{M}\right]_{ij}, \tag{23}$$

where δ_{ki} is the Kronecker delta, the partial derivative of $|\mathbf{X}|$ with respect to λ_i is

$$\frac{\partial |\mathbf{X}|}{\partial \lambda_i} = |\mathbf{X}| \sum_j \left[\mathbf{X}^{-T}\right]_{ij} \left[\mathbf{M}^T\mathbf{M}\right]_{ij}. \tag{24}$$

Finally, writing the expression above in compact form,

$$\boldsymbol{\nabla}_{\boldsymbol{\lambda}} |\mathbf{X}| = |\mathbf{X}| \mathbf{d}\big(\mathbf{X}^{-T}\mathbf{M}^T\mathbf{M}\big), \tag{25}$$

where $\mathbf{d}(\mathbf{A})$ denotes a column vector with the diagonal elements of matrix $\mathbf{A}$.

B. DERIVATIVE OF $R^B(\gamma, \mathrm{M})$ FOR THE 2×2 CASE

Defining

$$T = 2\lambda_1\lambda_2 m_{11} m_{12} m_{21} m_{22}, \tag{26}$$

$$S = 1 + \lambda_1 \left(m_{11}^2 + m_{21}^2\right) + \lambda_2 \left(m_{12}^2 + m_{22}^2\right) + \\ + \lambda_1\lambda_2 \left(m_{11}^2 m_{22}^2 + m_{12}^2 m_{21}^2\right), \tag{27}$$

for the particular case of a 2×2 MIMO system, in [7], it was found that

$$\mathbb{E}_{\mathbf{\Theta}} I\left(\mathbf{\Lambda}, \mathbf{H}\right) = \log\left(\frac{S + \sqrt{S^2 - T^2}}{2}\right) \triangleq G(\lambda_1, \lambda_2). \tag{28}$$

Using that $\lambda_1 + \lambda_2 = \gamma$ we can define $F(\lambda, \gamma) = G(\lambda, \gamma - \lambda)$. From its definition in (8) it can be seen that $R^B(\gamma, \mathbf{M}) = F(\lambda^\star, \gamma)$. Obviously $\lambda^\star = \lambda^\star(\gamma)$ and consequently

$$\frac{\mathrm{d}R^B}{\mathrm{d}\gamma} = \frac{\partial F}{\partial \lambda^\star}\frac{\mathrm{d}\lambda^\star}{\mathrm{d}\gamma} + \frac{\partial F}{\partial \gamma}, \tag{29}$$

which is valid only for

$$\gamma > \frac{\left(m_{11}^2 + m_{21}^2\right) - \left(m_{12}^2 + m_{22}^2\right)}{m_{11}^2 m_{22}^2 + m_{21}^2 m_{12}^2}. \tag{30}$$

If last condition is not met, it necessarily implies that $\lambda_1^\star = \gamma$ or $\lambda_2^\star = \gamma$ and the derivative is straightforward. See further [7].

The partial derivatives in (29) can be found straightforward from (26), (27), and (28). However, to calculate $\mathrm{d}\lambda^\star/\mathrm{d}\gamma$, we have to recall a result from [7] which relates $\lambda^\star$ with γ through a fourth-order polynomial $\pi(\lambda^\star, \gamma) = 0$. With that result now $\mathrm{d}\lambda^\star/\mathrm{d}\gamma$ can be found as

$$\frac{\mathrm{d}\lambda^\star}{\mathrm{d}\gamma} = -\frac{\partial \pi/\partial \gamma}{\partial \pi/\partial \lambda^\star}. \tag{31}$$

Using a similar approach it can also be found an expression for $\mathrm{d}^2 R^B/\mathrm{d}\gamma^2$ and after a little inspection it can be stated that $\gamma > 0$ implies that $-\mathrm{d}^2 R^B/\mathrm{d}\gamma^2 > 0$, but the calculations are not reproduced here because they are cumbersome.

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