ON THE CAPACITY OF LINEAR VECTOR GAUSSIAN CHANNELS WITH MAGNITUDE KNOWLEDGE AND PHASE UNCERTAINTY

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ABSTRACT

We are interested in studying different capacity formulations in linear vector Gaussian channels where the transmitter is only informed with the magnitude of the complex channel matrix coefficients and the receiver has perfect channel knowledge. Initially, we give the general expressions for the ergodic, compound, and outage capacities for our particular model of channel state information. Next, focusing on the compound formulation, we find that the optimal transmitter strategy consists in independent signaling through the transmit dimensions. Finally, we present a new result on the power allocation for the maximization of the outage mutual information.

1. INTRODUCTION

It is well known [1], that the capacity achieving signaling strategy for linear vector Gaussian channels consists in transmitting random Gaussian vectors with zero mean. Consequently, the only remaining degree of freedom to optimize the mutual information is through the covariance matrix of these vectors. The optimal design of the covariance matrix depends, to a large extent, on the model utilized to characterize the channel.

On one hand, many authors consider the linear vector channel as a stochastic matrix, whose entries are commonly modeled as circularly symmetric complex Gaussian random variables, with a given mean and covariance. Within this context, two different kind of approaches can be considered depending on the time-varying characteristics of the channel. If the channel fluctuations are fast enough so that, during the transmission of a message, its long term average properties are unveiled, then the optimal covariance design aims at maximizing the *ergodic* mutual information, because it is the measure that controls the rate at which reliable communication is possible [2]. On the contrary, in a slow fading scenario, the maximum rate achievable with a certain probability is dictated by the *outage* mutual information [3], and thus the covariance matrix should be optimized according to this criterion.

On the other hand, much attention is recently being paid to models that describe the channel assuming that it is a deterministic (thus fixed) quantity belonging to a certain set, which takes into account the possible effects of lack of knowledge about the channel state that the transmitter may be experiencing. Within this framework, the design of the covariance matrix is focused at maximizing the worst case mutual information, whose supremum is defined in [4] as the *compound* capacity.

Concerning the stochastic models for the channel, the ergodic capacity of multiantenna systems with partial channel state information at the transmitter side has been studied, for example, in [5] and [6], in which a single receiver architecture is considered, and it is assumed that the transmitter has access only to either the mean value or the covariance of the channel vector. In the latter case, capacity can be achieved by a covariance matrix that has the same eigenvectors as the true channel covariance matrix. The outage formulation has been largely less studied. In [2], it was conjectured that for Rayleigh channels the optimal strategy consists in performing uniform power allocation among a subset of the transmit dimensions. More recently, the outage capacity has been studied in [7] for the case of a single receive dimension. As far as the compound capacity is concerned, the optimality of the uniform power allocation scheme has been proven in [8] using game-theoretic justifications under a mild assumption on the channel isotropy. In addition, in [9] the authors proved that beamforming maximizes the compound capacity in rank one Ricean linear vector Gaussian channels. For the sake of completeness, see further [10] for a tutorial on capacity under different assumptions of the knowledge at the transmitter side.

In this work, we study the ergodic, compound and outage capacity formulations for the particular model of incomplete knowledge of the channel matrix at the transmitting end that was presented in [11, 12], where the transmitter is only informed of (or is only able to estimate accurately) the magnitude of the channel coefficients and lacks of information about the corresponding channel phases. The receiver is assumed to have perfect channel knowledge.

2. SYSTEM MODEL

We consider a linear vector Gaussian channel with n_T transmit and n_R receive dimensions. Let us define $\mathbf{x} \in \mathbb{C}^{n_T}$ as the transmit signal vector, $\mathbf{H} \in \mathbb{C}^{n_R \times n_T}$, as the channel matrix, and $\mathbf{n} \in \mathbb{C}^{n_R}$, as the noise vector, modeled as a circularly symmetric Gaussian distributed random vector with zero mean and covariance matrix $\mathbb{E}\mathbf{n}\mathbf{n}^\dagger = \sigma^2\mathbf{I}$. The received signal, $\mathbf{y} \in \mathbb{C}^{n_R}$, for this model can be expressed as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = [\mathbf{h}_1 \mathbf{h}_2 \cdots \mathbf{h}_{n_T}]\, \mathbf{x} + \mathbf{n}. \tag{1}$$

As it was stated in the introduction, we assume that, while $\mathbf{H}$ is perfectly known at the receiver, the transmitter only has access to the magnitude of the entries of $\mathbf{H}$. To separate the known from the unknown part of the elements of $\mathbf{H}$, we define

$$\mathbf{H} \equiv \mathbf{M} \odot \mathbf{P},$$

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where $\odot$ denotes the Hadamard element-wise matrix product. In this model, the magnitude information of the complex entries of $\mathbf{H}$ is stored in $\mathbf{M}$, and the coefficients of $\mathbf{P}$ contain the phase information, *i.e.*, $[\mathbf{P}]_{kl} = \mathrm{e}^{\mathrm{i}[\mathbf{\Theta}]_{kl}}$. Following the notation introduced above, the gain between the l-th transmit and the k-th receive dimensions is $[\mathbf{H}]_{kl} = [\mathbf{M}]_{kl}\mathrm{e}^{\mathrm{i}[\mathbf{\Theta}]_{kl}}$, with $[\mathbf{M}]_{kl} \geq 0, \forall k, l$. Whenever necessary, the unknown channel phases, $[\mathbf{\Theta}]_{kl}$, are modeled as i.i.d. uniform random variables in the interval $[-\pi, \pi)$ as suggested in, *e.g*, [13].

The capacity achieving strategy in linear vector Gaussian channels is signaling using random Gaussian vectors [1]. In this case, the mutual information for a fixed transmit covariance matrix and channel state is [2]

$$\Psi(\mathbf{Q}, \mathbf{M}, \mathbf{P}) = \log\det\left(\mathbf{I} + \sigma^{-2}(\mathbf{M}\odot\mathbf{P})\mathbf{Q}(\mathbf{M}\odot\mathbf{P})^{\dagger}\right), \quad (2)$$

where the transmitted power is given by $\mathbb{E}\mathbf{x}^{\dagger}\mathbf{x} = \operatorname{Tr}\mathbf{Q} = P$. In the following subsections, we particularize the definitions of ergodic, compound, and outage mutual information for the uncertainty model considered in this work, explain in which situations do these measures become meaningful, and give some previous known results.

2.1. Ergodic Mutual Information

The ergodic mutual information is defined in, *e.g.*, [10] as the expectation, with respect to the channel state uncertainty, of the mutual information expression in (2). In our case, it particularizes to

$$I_E(\mathbf{Q}, \mathbf{M}) = \mathbb{E}_{\mathbf{P}}\Psi(\mathbf{Q}, \mathbf{M}, \mathbf{P}). \quad (3)$$

The ergodic mutual information is a meaningful measure of the achievable rates in situations where, during the transmission of the message, the magnitude of the channel matrix, $\mathbf{M}$, remains constant while the channel phases, $\mathbf{P}$, vary sufficiently fast so that its long-term ergodic properties are revealed. This model would fit, *e.g.*, in a communication situation with direct line of sight, where the mobile user is moving slowly. In this scenario, while the magnitude of the entries of $\mathbf{M}$ would not change in an appreciable way (assuming plane waves propagation), the phases in $\mathbf{P}$ would vary very rapidly because of the relative movement.

The ergodic capacity, defined as the supremum of the ergodic mutual information with respect to the set of possible covariance matrices, subject to a mean transmitted power constraint, was found in [11] for this particular model of channel uncertainty as

$$\begin{aligned} C_E = \sup_{\mathbf{\Lambda}} &\; I_E(\mathbf{\Lambda}, \mathbf{M}) \\ \text{s.t.} &\; \operatorname{Tr}\mathbf{\Lambda} \leq P,\ \mathbf{\Lambda} \succcurlyeq \mathbf{0}, \end{aligned} \quad (4)$$

where $\mathbf{\Lambda}$ is a diagonal matrix. The optimal power allocation, *i.e.*, the entries of $\mathbf{\Lambda}$, were also found in [11] for a 2×2 channel case.

2.2. Compound Mutual Information

The compound mutual information is defined [4] as the infimum, with respect to the channel state uncertainty, of the mutual information expression in (2). In our case, it particularizes to

$$I_C(\mathbf{Q}, \mathbf{M}) = \inf_{\mathbf{P}\in\mathcal{P}} \Psi(\mathbf{Q}, \mathbf{M}, \mathbf{P}), \quad (5)$$

where $\mathcal{P} \equiv \left\{\mathbf{X} \in \mathbb{C}^{n_R\times n_T} \mid |[\mathbf{X}]_{ij}| = 1\right\}$ defines the set of all possible channel phases compatible with the incomplete knowledge about $\mathbf{H}$. Noticeworthy, the compound mutual information does not depend on the statistical properties of the uncertainty in the channel, because it only considers the worst-case scenario.

The compound mutual information is a measure of the worst-case achievable rates in situations where no significant channel variability may occur during the transmission of the message and the transmitter is only informed of (or is only able to estimate accurately) the magnitude matrix $\mathbf{M}$. This may be the case of a static communication between the transmitter and the receiver, or when communicating in a slow fading environment.

In Section 3 we give the structure of the covariance matrix that maximizes the compound mutual information, for our particular case of channel knowledge, which turns out to be a diagonal structure. In [12], the authors imposed a diagonal structure on the covariance matrix, and found the optimal power allocation for the compound mutual information, for the case of a $2 \times n_R$ channel and also for a general $n_T \times n_R$ channel configuration in the low signal-to-noise ratio (SNR) regime.

2.3. Outage Capacity

The outage mutual information [3] is the maximum rate, R, such that the probability that the mutual information in (2) be below R is less or equal than ϵ as expressed in (6).

$$I_O^{\epsilon}(\mathbf{Q}, \mathbf{M}) = \sup_R \left\{R \,|\, \Pr\left(\Psi(\mathbf{Q}, \mathbf{M}, \mathbf{P}) \leq R\right) \leq \epsilon\right\}, \quad (6)$$

where the probability is with respect to the distribution of the channel phases in $\mathbf{P}$. The outage mutual information can be technologically relevant in static scenarios, or when the channel fluctuations are slow enough so that the channel can be considered fixed[1] during the transmission of the message. From its expression in (6), we can see that the outage mutual information can be directly related to the achievable rate that can be guaranteed with a certain probability, which is given by $1 - \epsilon$.

The outage capacity problem can be stated from two different approaches, which are essentially equivalent. The problem is to find the optimal covariance matrix to, on one hand, given an outage probability, obtain the corresponding maximum rate; or, on the other hand, given a target rate, obtain its associated minimum outage probability. Considering the latter point of view, for a target rate C_{out}, which may depend on P and σ^2, we obtain the following problem formulation for the minimum outage probability ϵ_{min},

$$\begin{aligned} \epsilon_{\text{min}}(C_{\text{out}}) = \inf_{\mathbf{Q}} &\; \Pr\left(\Psi(\mathbf{Q}, \mathbf{M}, \mathbf{P}) \leq C_{\text{out}}\left(P, \sigma^2\right)\right) \\ \text{s.t.} &\; \operatorname{Tr}\mathbf{Q} \leq P,\ \mathbf{Q} \succcurlyeq \mathbf{0}. \end{aligned} \quad (7)$$

Unfortunately, for the outage formulation no results are known for our channel uncertainty model. In Section 4 we impose a diagonal structure to the covariance matrix $\mathbf{Q}$ in (7) and find a solution, that gives an upper bound on ϵ_{min}.

3. OPTIMAL COVARIANCE MATRIX FOR THE COMPOUND MUTUAL INFORMATION

The compound capacity of a linear vector Gaussian channel where the transmitter has access only to the amplitude of the channel matrix coefficients and has phase uncertainty is defined [4] as the constrained optimization of the compound mutual information in (5) with respect to the covariance matrix $\mathbf{Q}$, *i.e.*,

$$\begin{aligned} C_C(\mathbf{M}) = \sup_{\mathbf{Q}} &\; I_C(\mathbf{Q}, \mathbf{M}) \\ \text{s.t.} &\; \operatorname{Tr}\mathbf{Q} \leq P,\ \mathbf{Q} \succcurlyeq \mathbf{0}. \end{aligned} \quad (8)$$

[1] The channel is considered fixed, but the transmitter has only partial (or no) knowledge of the channel state matrix.

We are interested in finding the structure of the covariance matrix $\mathbf{Q}$ that is the solution to the problem in (8). To accomplish this purpose, we first present the following lemmas.

Lemma 1. *Let $\mathbf{J}_n \in \mathbb{Z}^{n_T \times n_T}$ be a diagonal matrix such that its non-zero entries satisfy $[\mathbf{J}_n]_{ii} \in \{1, -1\}$, $\forall i$. There are $L = 2^{n_T}$ different such matrices which are indexed from $n = 1$ to $n = L$. It then follows that*

$$\mathbf{\Lambda} = \frac{1}{L} \sum_{n=1}^{L} \mathbf{J}_n \mathbf{Q} \mathbf{J}_n \qquad (9)$$

is a diagonal matrix such that $[\mathbf{\Lambda}]_{ii} = [\mathbf{Q}]_{ii}$, $\forall \mathbf{Q} \in \mathbb{C}^{n_T \times n_T}$.

Lemma 2. *The function $I_C(\mathbf{Q}, \mathbf{M})$ is invariant under the transformation $\mathbf{Q} \mapsto \mathbf{J}_n \mathbf{Q} \mathbf{J}_n$, $\forall n$, where the definition of $\mathbf{J}_n$ is the same as in Lemma 1.*

Proof. We begin by noticing that

$$(\mathbf{M} \odot \mathbf{P}) \mathbf{J}_n \mathbf{Q} \mathbf{J}_n (\mathbf{M} \odot \mathbf{P})^\dagger = (\mathbf{M} \odot \widetilde{\mathbf{P}}) \mathbf{Q} (\mathbf{M} \odot \widetilde{\mathbf{P}})^\dagger, \qquad (10)$$

where we have introduced the following change of variables

$$[\widetilde{\mathbf{P}}]_{ij} = \begin{cases} [\widetilde{\mathbf{P}}]_{ij} & \text{if } [\mathbf{J}_n]_{jj} = 1 \\ [\widetilde{\mathbf{P}}]_{ij}\, \mathrm{e}^{\mathrm{i}\pi} & \text{if } [\mathbf{J}_n]_{jj} = -1 \end{cases}.$$

Notice that $\mathbf{P} \in \mathcal{P} \Rightarrow \widetilde{\mathbf{P}} \in \mathcal{P}$. In (11), we perform the transformation $\mathbf{Q} \mapsto \mathbf{J}_n \mathbf{Q} \mathbf{J}_n$ on $I_C(\mathbf{Q}, \mathbf{M})$ and show that it remains invariant.

$$\begin{aligned} I_C(\mathbf{Q}, \mathbf{M}) &\mapsto I_C(\mathbf{J}_n \mathbf{Q} \mathbf{J}_n, \mathbf{M}) = \\ &= \inf_{\mathbf{P} \in \mathcal{P}} \log\det\left(\mathbf{I} + (\mathbf{M} \odot \mathbf{P}) \mathbf{J}_n \mathbf{Q} \mathbf{J}_n (\mathbf{M} \odot \mathbf{P})^\dagger\right) \\ &= \inf_{\widetilde{\mathbf{P}} \in \mathcal{P}} \log\det\left(\mathbf{I} + (\mathbf{M} \odot \widetilde{\mathbf{P}}) \mathbf{Q} (\mathbf{M} \odot \widetilde{\mathbf{P}})^\dagger\right) \qquad (11) \\ &= I_C(\mathbf{Q}, \mathbf{M}), \end{aligned}$$

where last equality follows from the fact that $\widetilde{\mathbf{P}}$ is a dummy minimization variable. □

Lemma 3. *The function $I_C(\mathbf{Q}, \mathbf{M})$ is strictly concave in $\mathbf{Q}$, where $\mathbf{Q}$ belongs to the set of positive semidefinite matrices.*

Proof. The function $I_C(\mathbf{Q}, \mathbf{M})$ is defined in (5) as the pointwise infimum, with respect to $\mathbf{P} \in \mathcal{P}$, of a set of strictly concave functions in $\mathbf{Q}$. Consequently, from [14, p. 81], $I_C(\mathbf{Q}, \mathbf{M})$ is also strictly concave. □

In the following proposition, we find the structure of the covariance matrix that is the solution to the problem in (8).

Proposition 1. *The compound capacity of a linear vector Gaussian channel, where the receiver has perfect channel state information and the transmitter is only informed with the amplitude of the channel complex coefficients, can be achieved by (and only by) a Gaussian codebook with a diagonal covariance matrix.*

Proof. To prove achievability we only need to show that

$$\begin{aligned} I_C(\mathbf{Q}, \mathbf{M}) &= \sum_{n=1}^{L} \frac{1}{L} I_C(\mathbf{Q}, \mathbf{M}) && (12) \\ &= \sum_{n=1}^{L} \frac{1}{L} I_C(\mathbf{J}_n \mathbf{Q} \mathbf{J}_n, \mathbf{M}) && (13) \\ &\leq I_C\left(\frac{1}{L} \sum_{n=1}^{L} \mathbf{J}_n \mathbf{Q} \mathbf{J}_n, \mathbf{M}\right) && (14) \\ &= I_C(\mathbf{\Lambda}, \mathbf{M}), && (15) \end{aligned}$$

where $\mathbf{\Lambda}$ is a diagonal matrix such that $\operatorname{Tr}\mathbf{\Lambda} = \operatorname{Tr}\mathbf{Q}$. Note that (13) follows from Lemma 2; inequality (14), from Lemma 3; and, finally, we invoked Lemma 1 to write (15). It is now straightforward to see that, if $\mathbf{Q}$ is a solution of the optimization problem in (8), then there exists a diagonal matrix $\mathbf{\Lambda}$ that is also optimal or, formally,

$$\sup_{\mathbf{Q}} I_C(\mathbf{Q}, \mathbf{M}) \leq \sup_{\mathbf{\Lambda}} I_C(\mathbf{\Lambda}, \mathbf{M}),$$

where $\mathbf{\Lambda}$ is a diagonal matrix such that $\operatorname{Tr}\mathbf{\Lambda} = \operatorname{Tr}\mathbf{Q}$. □

Proof. To prove the converse we use a similar argument as in [15]. Since $I_C(\mathbf{Q}, \mathbf{M})$ is strictly concave in $\mathbf{Q}$ it has a unique global maximum, $\mathbf{Q}^\star$. From Lemma 2, we deduce that, for $\mathbf{Q}^\star$ to be unique, it has to satisfy $\mathbf{Q}^\star = \mathbf{J}_n \mathbf{Q}^\star \mathbf{J}_n$, $\forall n$, which implies that $\mathbf{Q}^\star$ has to be diagonal. □

Interestingly, the result in Proposition 1 for the compound case also holds for the ergodic case. It can be shown just by replacing $I_C(\mathbf{Q}, \mathbf{M})$ by $I_E(\mathbf{Q}, \mathbf{M})$ in the formulation presented above. Notice that the lemmas that we used in the derivation are also valid for the ergodic mutual information, however, we do not give the proofs here because the optimality of a diagonal covariance matrix for the ergodic capacity case was already proved in [11]. Unfortunately, the same result does not apply for the outage capacity because the objective function in (7), is not concave in $\mathbf{Q}$ and consequently Lemma 3 is not valid for the outage formulation.

4. TWO TRANSMITTERS CASE

In this section, we consider the particular case where the transmitting end is equipped with two antennas and the receiver collects the incoming signal through a large number of antennas. In addition, we consider that the 2×2 transmit covariance matrix is diagonal, with entries λ_1 and λ_2, which, we recall, is optimal in the ergodic, and compound formulations, but not necessarily in the outage case, obtaining, thus, an upper bound on the minimum probability in (7).

Particularizing the expression in (2) for the $2 \times n_R$ case, with a diagonal covariance we obtain

$$\Psi = \log\left((1 + \lambda_1 M_1)(1 + \lambda_2 M_2) - \lambda_1 \lambda_2 |M_{12}|^2\right), \qquad (16)$$

where now $\lambda_1 + \lambda_2 \leq P/\sigma^2 \triangleq \gamma$ and where $M_1 = \|\mathbf{h}_1\|^2$, $M_2 = \|\mathbf{h}_2\|^2$, and $M_{12} = \mathbf{h}_1^\dagger \mathbf{h}_2$. We assume w.l.o.g. that $M_1 > M_2$ as the opposite case is symmetric, and we would only need to interchange the roles of λ_1 and λ_2. Notice that M_{12} is the only term in (16) that depends on the channel phases. Consequently, if any $\lambda_i = 0$ then the mutual information in (16) becomes a deterministic quantity. The pdf of M_{12} as a function of the amplitude of the channel coefficients can be rather complicated to deal with, so we assume that n_R is large enough and approximate it for a Gaussian distribution.[2] Thus, we obtain $M_{12} \sim \mathcal{CN}\left(0, \varsigma^2 = \sum_{i=1}^{n_R} |[\mathbf{H}]_{i2} [\mathbf{H}]_{i1}|^2\right)$. Consequently, $2|M_{12}|^2/\varsigma^2$ will be a χ^2 random variable with two degrees of freedom, and the expression in (16) can be very well approximated by

$$\Psi \approx \widetilde{\Psi} = \log\left((1 + \lambda_1 M_1)(1 + \lambda_2 M_2) - \frac{\lambda_1 \lambda_2 \varsigma^2}{2} \chi^2\right). \qquad (17)$$

Last equation is utilized to approximate the mutual information function in the ergodic (3) and outage (6) expressions, which are then used as objective functions in the optimization problems (4) and (7),

[2] For practical purposes $n_R \geq 8$ is enough for the majority of cases.

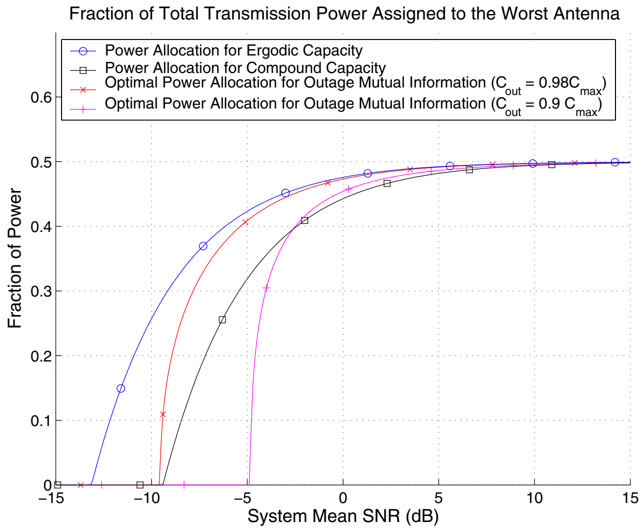

Fig. 1. Optimal power allocation for different criteria. In the case of outage mutual information we have considered two cases $C_{\text{out}} = 0.98 C_{\max}(\gamma)$ and $C_{\text{out}} = 0.9 C_{\max}(\gamma)$, where $C_{\max}$ is the supremum with respect of λ_1 and λ_2 of the expression in (16) when $M_{12} = 0$. Notice that, in this case, $C_{\max}$ is a function of γ (mean SNR).

respectively. In each case, letting $\lambda_1 = \lambda$, $\lambda_2 = \gamma - \lambda$, a single dimensional problem in λ is obtained. We highlight that in the compound case, no approximation need to be done since the optimal power allocation expression was found in [12] in exact closed form.

In the ergodic case, the problem in (4) has to be solved numerically by a simple linear search in $\lambda \in [0, \gamma]$. Here, we give the solution for the power allocation that maximizes the outage mutual information problem expressed in (7).

If $\log(1 + \gamma M_1) > C_{\text{out}}$, then the solution to the outage mutual information is simply given by $\lambda = \gamma$, because in that case, the expression in (16) is not a random variable any more and then $\epsilon_{\min}(C_{\text{out}}) = 0$, which is always a solution of (7). Otherwise, if $\log(1 + \gamma M_1) \leq C_{\text{out}}$, then, defining $\zeta = \mathrm{e}^{C_{\text{out}}}$, the solution is

$$\lambda = \frac{\gamma M_2 + 1 - \zeta + \sqrt{(1 - \zeta + \gamma M_1)(1 - \zeta + \gamma M_2)}}{M_2 - M_1}.$$

To see the differences among the solution of each power allocation strategy, we have considered a 2×8 linear vector Gaussian channel. In Fig. 1 we have plotted the obtained optimal power allocation for the cases of ergodic, compound, and outage capacities. Note that, as expected, as the SNR increases, the solutions of all these three cases tend to the uniform power allocation, and, as the SNR decreases, all of them tend to give no power to the worst antenna (best and worst here is in terms of $M_1 \lessgtr M_2$). The other remark of interest, is the moment, in terms of the SNR, where a particular strategy (ergodic, compound, or outage), begins to allocate some power to the worst antenna, which we shall denote as γ_0. For the sake of space, we do not derive the expressions here, and just give their expressions. They read as

$$\gamma_0^E \approx \frac{M_1 - M_2}{M_1 M_2}, \quad \gamma_0^C = \frac{M_1 - M_2}{M_1 M_2 - M_X^2}, \quad \gamma_0^O = \frac{\mathrm{e}^{C_{\text{out}}(\gamma_0^O)} - 1}{M_1},$$

where $M_X = \sum_{i=1}^{n_T} |[\mathbf{H}]_{i1}[\mathbf{H}]_{i2}|$. Clearly, $\gamma_0^E < \gamma_0^C$ which agrees with the results plotted in Fig. 1. In addition, notice that γ_0^O has to be solved iteratively, and that its value depends on the particular choice of the function $C_{\text{out}}(\gamma)$ (also in accordance with the plots in Fig. 1).

5. CONCLUSION

In this work, we have shown that, in a linear vector Gaussian channel where the transmitter has only access to the magnitude of the channel matrix coefficients, the compound capacity can only be achieved by independent Gaussian signaling through the transmit dimensions, *i.e.*, the optimal covariance matrix is diagonal.

With this result we have gone a step further to the full characterization of the optimal covariance matrix, since the elements of the diagonal of the covariance matrix were already calculated in [12] for different channel configurations, but there, the diagonal structure was imposed, without proving its optimality.

Finally, for the particular case of two transmitters, we have obtained the resulting power allocations when maximizing the ergodic, compound, and outage mutual informations, which, as we have seen, become meaningful measures of the achievable rates depending on the characteristics of the underlying communication scenario.

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