Proving almost anything

A beginner's guide to a world with infinite solutions

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Professors frequently have to prove the truth of intuitions they have. When they are unable to do so, they tell their graduate students to prove them. (They, themselves, are too busy.) However, the most clever professors place these theorems on problem sets and exams. If any students actually manage to prove these "intuitions," the professors immediately publish the proofs to help secure their tenure bids. So, whether you are a professor or a graduate student, you have undoubtedly needed to prove something which seemed exceedingly difficult to prove.

Countless hours have been wasted attempting to prove these theorems. This is because the individual did not have access to a compendium of tried and true techniques. Thus, a list providing proof techniques at your disposal is essential. But there is another even more important reason. Sometimes you will be called on to prove a theorem which is simply not true. For these more troublesome proofs, standard methods just won't work. You need a special set of proof methods. So, whether you are seeking short cuts or methods to prove those troublesome theorems, these techniques can help.

Methods of confusion

Proof by obfuscation

If you write with the intent of confusing the reader, you may succeed. Since the grader cannot pinpoint where your answer deviates from the correct answer, she cannot mark your answer "wrong."

Proof by handwaving

Very similar to obfuscation, but this approach at least attempts to SOUND like a proof.

Proof by jargon

If you write as if you know what you are talking about, you may convince the reader that you are more knowledgeable than he is. He will then be afraid to disagree or mark your answer "wrong."

Proof by illegibility

If the grader cannot read your answer, how can it be wrong?

Proof by saturation

If you write a many-page "proof," no one will bother to check.

Proof by condensation

The opposite of saturation. You write an extremely terse proof in which you skip many steps (thereby avoiding the necessity of proving them!).

Methods of wishful thinking

Proof by intuition

If it feels good, go with it.

Proof by reputation

Any idea you have ever had has already been thought of by Ken Arrow. Since Arrow is always right, your idea must be true too. (For non-economists, Kenneth Arrow is a Nobel laureate at Stanford University (CA) whose intuitions are better than most people's theorems.)

Proof by prayer

If you pray that something is true, God might answer your prayer.

Proof by inebriation (or hallucination)

Everything always looks better when you're drunk.

Proof by self-deprecation

Think to yourself: "I've proven this to be false, but I am ALWAYS wrong, so it MUST be true!"

Proof by analogy

"A quasi-concave function is LIKE a concave function, so..."

Proof by exasperation

If you stare at a partial, incorrect proof long enough, it slowly looks more and more like a complete, correct proof.

Proof by assumption

If your assumptions are invalid, then your results are invalid too, so you might as well assume what you want to prove. This saves time and frustration.

Proof by consensus

If everyone in the study group agrees, then it's got to be true!

Proof by self-flagellation

Hit your head until your brain comes around to your way of thinking.

Proof by intimidation

Threaten the listener/reader/grader with serious consequences if she or he disagrees (or speak in a threatening manner).

Proof by trivialization

How many times have you read in a textbook, "It is obvious that..." or "It can easily be shown that..." or, "The attentive reader will understand that..." It is no coincidence that these statements are always made about the

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most difficult concepts in the book. The author(s) could not prove them, so they trivialized them. It's a very effective technique with widespread application and makes you look very, very smart in the process!

Proof by supposition
Suppose that the statement you are trying to prove is true. Try to show a contradiction. If you cannot, then the statement must be true!

Proof by repetition
If you repeat the result over and over again in your head, it begins to sound true.

Methods of cheating

Proof by upper-year students
Consult your favorite upper-year student who has been through the course already.

Proof by last year's answer set
Superior to consulting upper-year students, but only useful if they have bothered to save their notes. The more confusing the class, the more likely they are to have burned their notes.

Proof by proof
Just find the result in a book somewhere and copy it.

Methods of transformation

If you get a result that you don't like, there are a few special techniques you can use to transform your intermediate result into a more desirable final result. These methods are applicable only in special cases, but when they work, they can be a powerful method of proof.

Proof by interpolation
If you do two proofs and get two different answers and the answer that you want lies in between, just take the appropriate convex combination.

Proof by parentheses
Let's say that you have reduced the proof to solving the equation $3 + 14 + 1 = 20$. It may seem that you are stuck, but if you are observant, you will notice that adding parentheses will solve the problem: $(3 + 1)(4+1) = 20$.

Proof by transformation
This encompasses a few cases. If you get $X=2$ and you need $X= 10$, just switch to base 2. Or, if you get $X=0$ and you need $X=1$, just exponentiate. (These may seem to have limited application, but since many proofs yield $X=0$ or $X=1$, the ability to switch between them by taking logs or antilogs is a very useful technique!)

Proof by sign change
Just multiply one side of the equation by 1.

Proof by elimination
Just cross out any troublesome terms. If you work for hours to derive a final equality saying something like "x = 2x.," then breathe easy because you're very, very close to the correct answer. Just realize that you probably made a multiplication error. Go ahead and cross off the 2.

Proof by trace operator
Let's imagine that you're trying to show that the assumptions of your theorem lead to an identity and you get down to the equation $XYX^{-1} = Y$. This looks intractable, but if you remember to use the trace operator you're done because: $tr(XYX^{-1}) = tr(YX^{-1}X) = tr(YI) = tr(Y)$

Proof by conveniently ignoring the matrix conformability problem
Many people have already discovered this method and derived miraculously simple proofs, but it's such a useful technique that it must be listed in this compendium. Imagine that you need to show that $X'X(XX')^{-1} = I$ when X is not a square matrix. A lot of very smart people give up on this problem very quickly. But if you stare at it long enough, you will realize that if you just ignore the matrix conformability problem, it's basically true since $X'X$ is just the reverse of XX' and both are symmetric, square matrices.

Methods of delay

Proof by maturation
Just tell the grader that you will prove this later when you are older and wiser.

Proof by Fermat
"I had an elegant little proof of this, but this paper is not large enough to write it down..."

Proof by time constraint
When you absolutely must turn in the problem set, your pseudo answer suddenly becomes a full proof. You just scribble it down and you're done. [*Corollary*: Wait until the last minute! You save lots of time this way.]

About the author

James Lavin is a Ph.D. student in Economics at Stanford University (CA). He attended Harvard College and has an M.Sc. in Economics from the London School of Economics. He has recently written papers on the impact of the 1992 presidential election on stock prices and on the effect of minimum wage increases on employment in the fast food industry. He also studies state government budgeting, Congress, and economic reform in China. This past June, he married Ms. Yingmei Zhu, a physics Ph.D. student at Stanford from the People's Republic of China. His email address is lavin@leland.stanford.edu.