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Flexible Design of Cognitive Radio Wireless Systems

From game theory to variational inequality theory

Game theory is a field of applied mathematics that describes and analyzes scenarios with interactive decisions. In recent years, there has been a growing interest in adopting cooperative and noncooperative game theoretic approaches to model many communications and networking problems, such as power control and resource sharing in wireless/wired and peer-to-peer networks and routing/flow control in communication networks. A more general framework suitable for investigating and solving various equilibrium models, even when classical game theory may fail, is known to be the variation inequality (VI) problem that constitutes a very general class of problems in nonlinear analysis. The goal of this article is to show how many challenging unsolved resource allocation problems in the emerging field of cognitive radio (CR) networks fit naturally either in the game theoretical paradigm or in the more general theory of VI. This provides us with all the mathematical tools necessary to analyze the proposed equilibrium problems for CR systems (e.g., existence and uniqueness of the solution) and to devise distributed algorithms along with their convergence properties.

MOTIVATION

Recently, the increasing demand of wireless services has made the radio spectrum a very scarce and precious resource. Moreover, most current wireless networks characterized by fixed spectrum assignment policies are known to be very inefficient considering that licensed bandwidth demands are highly varying along the time and/or space dimensions. Indeed, according to the Federal Communications Commission (FCC), only 15–85% of the licensed spectrum is utilized on the average [1]. CR originated as a possible solution to this problem [2] obtained by endowing the radio nodes with "cognitive capabilities," e.g., the ability to sense the electromagnetic environment, make short-term predictions, and react consequently by adapting transmission parameters (e.g., operating spectrum, modulation, and transmission power) to optimize the

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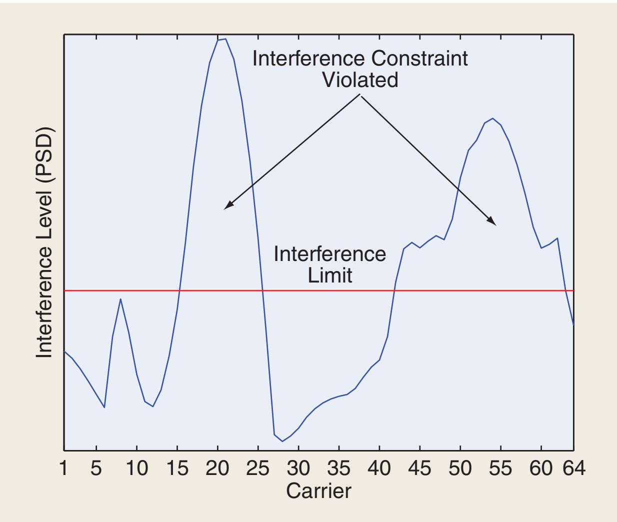
usage of the available resources [3]–[5]. The widely accepted debated position proposed for implementing the spectrum sharing idea of CR calls for a hierarchical access structure, distinguishing between primary users, or legacy spectrum holders, and secondary users, who access the licensed spectrum dynamically, under the constraint of not inducing intolerable quality of service (QoS) degradations on the primary users [3]–[5]. Within this context, alternative approaches have been considered to allow concurrent communications (see [5] for a recent tutorial on the topic).

In this article, we focus on opportunistic resource allocation techniques in hierarchical cognitive networks (also known in the CR literature as interweave communications [5]), as they seem to be the most suitable for the current spectrum management policies and legacy wireless systems [4]. In particular, our interest is in devising the most appropriate form of concurrent communications of cognitive users competing over the physical resources that primary users make available. Looking at the opportunistic communication paradigm from a broad signal processing perspective, the secondary users are allowed to transmit over a multidimensional space, whose coordinates may represent time slots, frequency bins, and (possibly) angles, with the goal of finding out the most appropriate transmission strategy exploring all available degrees of freedom, under the constraint of inducing a limited interference, or no interference at all, on the primary users.

One approach to devise such a system design would be using global optimization techniques, under the framework of network utility maximization (NUM) (see, e.g., [6]). However, recent results in [7] have shown that the NUM problem based on the maximization of the information rates over frequency-selective single-input, single-output (SISO) interference channels is an NP-hard problem, under different choices of the system utility function. Consequently, several attempts have been pursued in the literature to deal with the nonconvexity of such a problem. Some works proposed suboptimal or closed-to-optimal algorithms based on duality theory (see, e.g., [8]–[9]). Others works applied the theory of cooperative games (based on the Nash bargaining optimality criterion) to compute, under technical conditions and/or simplifying assumptions on the users' transmission strategies, the largest achievable rate region of the system. Two good tutorials on the topic are [10]–[11], published in this special issue. However, current algorithms based on global optimization or Nash bargaining solution lack any mechanism to control the amount of aggregate interference generated by the transmitters. Moreover, they are centralized and computationally expensive. This raises some practical issues that are insurmountable in the CR context. For example, these algorithms need a central node having full knowledge of all the channels and interference structure at every receiver, which poses serious implementation problems in terms of scalability and amount of signaling to be exchanged among the nodes. For these reasons, in this article, we follow a different approach and we concentrate on decentralized strategies, where the cognitive users are able to self-enforce the negotiated agreements on the usage of the available spectrum without the intervention of a centralized authority. The philosophy underlying this approach is a competitive optimality criterion, as every user aims for the transmission strategy that unilaterally maximizes his own payoff function. This form of equilibrium is, in fact, the well-known concept of Nash equilibrium (NE) in game theory (see, e.g., [12], [13]).



[FIG1] Power spectral density of the interference profile at the receiver of a primary user generated by the classical iterative waterfilling algorithm.

Because of the inherently competitive nature of multiuser systems, it is not surprising indeed that game theory has been already adopted to solve distributively many resource allocation problems in communications. An early application of game theory in a communication system is [14], where the information rates of the users were maximized with respect to the power allocation in a DSL system modeled as a frequency-selective (in practice, multicarrier) Gaussian interference channel. Extension of the basic problem to ad-hoc frequency-selective and multiple-input, multiple-output (MIMO) networks were given in [15]–[19] and [20]–[23], respectively. However, the results in the cited papers have been recognized not to be applicable to CR systems because they do not provide any mechanism to control the amount of interference generated by the secondary users on the primary users [3]. Figure 1 shows an example of the interference profile generated over a given portion of spectrum by the classical iterative waterfilling algorithm (IWFA) [14], [24], and [25] at the receiver of a primary user that can only tolerate a maximum interference as indicated. The interference received by the primary user can be arbitrary large and the interference constraint (also called temperature-interference constraint) is not satisfied.

In this article, we fill this gap and provide an overview of different equilibrium problems suitable to design cognitive (possibly) MIMO transceivers within the paradigm of opportunistic communications. We show how game theory and the more general theory of VI can be successfully applied to solve some design challenges in CR systems. Our main results are: i) the establishment of conditions guaranteeing that the dynamic interactions among cognitive nodes admit a (possibly unique) equilibrium solution, under different interference-controlled paradigms preserving the

QoS of the primary users; and ii) practical and efficient algorithms able to reach the equilibrium points, with no coordination among the secondary users. The proposed algorithms differ in performance, level of protection of the primary users, computational effort and signaling among primary and secondary users, convergence analysis, and convergence speed; which makes them suitable for many different CR systems.

NONCOOPERATIVE GAMES AND VI PROBLEMS: BASIC CONCEPTS

We first introduce some basic definitions and concepts from the theory of static noncooperative games and variational inequalities that are used in the article. The literature is enormous; we refer the interested reader to [12]–[13] and [26] as entry points, and [27] for more advanced results.

NONCOOPERATIVE GAMES

A noncooperative strategic form game, also called an NE problem (NEP), models a game where all players act independently and simultaneously according to their own self-interests and with no a priori knowledge of other players strategies. Stated in mathematical terms, a static $Q-$ player game in strategic form is a triplet $\mathcal{G} = \langle \Omega, \mathcal{Q}, \mathbf{u} \rangle$ composed of a set of players $\Omega = \{1, 2, \ldots, Q\}$, a set of possible combinations of actions of each player (called admissible strategy set) denoted by $\mathcal{Q} = \prod_{i=1}^{Q} \mathcal{Q}_i$, where $\mathcal{Q}_i$ is the set of actions for the ith player, and a vector utility function $\mathbf{u} = (u_i)_{i=1}^{Q}$, where $u_i(\mathbf{x}) : \mathcal{Q} \to \mathbb{R}$ is the utility of the ith player that depends in general on the strategies $\mathbf{x} = (\mathbf{x}_i, \mathbf{x}_{-i})$ of all players; $\mathbf{x}_i \in \mathcal{Q}_i$ denotes a feasible strategy profile of player i, and $\mathbf{x}_{-i} = (\mathbf{x}_j)_{j \neq i}$ is a vector of strategies of all players except i. Similarly, we denote by $\mathcal{Q}_{-i} = \prod_{j \neq i} \mathcal{Q}_j$ the joint strategy set of all players except i. The interpretation of u_i is that player i receives a payoff of $u_i(\mathbf{x}_1, \ldots, \mathbf{x}_Q)$ once the players have chosen the strategies $\mathbf{x}_1, \ldots, \mathbf{x}_Q$. Our interest in this article is focused on the Nash game in the form described above, where each player has a set of strategies that is independent from the actions of the other players. However, in reality sometimes the actions of the players are constrained by the actions of the other players. The so-called generalized NE problem (GNEP) introduced by Arrow and Debreu captures exactly this dependence (see, e.g., [28] for details). Recently, the GNEP has been successfully applied to model and solve certain power control games over frequency-selective interference channels. An overview of the state-of-the-art results based on the framework developed in [19] can be found in [11], also published in this special issue.

The noncooperative paradigm postulates the rationality in the behaviors of the players: Each player i competes against the others by choosing a strategy profile $\mathbf{x}_i \in \mathcal{Q}_i$ that maximizes his payoff function $u_i(\mathbf{x}_i, \mathbf{x}_{-i})$, given the actions $\mathbf{x}_{-i} \in \mathcal{Q}_{-i}$ of the other players. A static noncooperative game in strategic form can be then represented as a set of coupled optimization problems

$$(\mathcal{G}): \quad \begin{array}{ll} \underset{\mathbf{x}_i}{\text{maximize}} & \mathrm{u_i}(\mathbf{x}_i, \mathbf{x}_{-i}) \\ \text{subject to} & \mathbf{x}_i \in \mathcal{Q}_i, \end{array} \qquad \forall i \in \Omega. \tag{1}$$

Player i's problem in (1) is to determine, for each fixed but arbitrary tuple $\mathbf{x}_{-i}$ of the other players' strategies, an optimal strategy $\mathbf{x}_i^*$ that solves the maximization problem in the variable $\mathbf{x}_i \in \mathcal{Q}_i$.

A desirable solution to (1) is one in which individual (rational) players act in accordance with their incentives, maximizing their own payoff function. This idea is best captured by the notion of NE: An action profile $\mathbf{x}^* \in \mathcal{Q}$ of game $\mathcal{G}$ is a pure-strategy NE if the following condition holds for all $i \in \Omega$:

$$u_i(\mathbf{x}_i^*, \mathbf{x}_{-i}^*) \geq u_i(\mathbf{x}_i, \mathbf{x}_{-i}^*), \ \forall \mathbf{x}_i \in \mathcal{Q}_i. \tag{2}$$

In words, an NE is a (self-enforcing) strategy profile, with the property that no single player can unilaterally benefit from a deviation from it, given that all the other players act according to it.

It is useful to restate the definition of NE in terms of a fixedpoint solution to the best response mapping $\mathcal{B}(\mathbf{x}) = \mathcal{B}_1(\mathbf{x}_{-1}) \times \mathcal{B}_2(\mathbf{x}_{-2}) \times \cdots \times \mathcal{B}_Q(\mathbf{x}_{-Q})$, where each $\mathcal{B}_i(\mathbf{x}_{-i})$ is the set of the optimal solutions to the ith optimization problem in (1), given $\mathbf{x}_{-i} \in \mathcal{Q}_{-i}$. It is not difficult to see that a strategy profile $\mathbf{x}^* \in \mathcal{Q}$ is a pure strategy NE of $\mathcal{G}$ if and only if $\mathbf{x}^*$ is a fixed point of $\mathcal{B}(\mathbf{x})$, i.e., $\mathbf{x}^* \in \mathcal{B}(\mathbf{x}^*)$. This alternative formulation of the equilibrium solution may be useful to address some essential issues of the equilibrium problems, such as the existence and uniqueness of solutions, stability of equilibria, and design of effective algorithms for finding equilibrium solutions, thus paving the way to the application of the fixed-point machinery. In fact, in general, the uniqueness or even existence of a pure strategy NE is not guaranteed; neither is convergence to an equilibrium when one exists. Sometimes, however, the structure of a game is such that one is able to establish one or more of these desirable properties, as for example happens in potential games [29] or supermodular games [30], which have recently received great attention in the signal processing and communication communities as a useful tool to solve various power control problems in wireless communications and networking [31]–[33].

The study of the existence of pure strategy equilibria under weaker and weaker assumptions and the analysis of some structural properties of the solutions, such as uniqueness or local uniqueness, have been investigated extensively in the literature. An overview of the relevant literature is [28]. For the purpose of this article, it is enough to recall an existence result that is one of the simplest of the genre, based on the interpretation of the NE as fixed point of the best-response mapping and existence results from fixed-point theory (Kakutani fixed-point theorem). A pure strategy NE exists for a game $\mathcal{G} = \langle \Omega, \mathcal{Q}, \mathbf{u} \rangle$ if: i) each player's strategy set $\mathcal{Q}_i$ is convex and compact (i.e., closed and bounded); and ii) the payoff function $u_i(\mathbf{x}_i, \mathbf{x}_{-i})$ of each player is continuous in $\mathbf{x}$ and quasi-concave in $\mathbf{x}_i$, for any fixed $\mathbf{x}_{-i}$ (note that concavity of each $u_i(\mathbf{x}_i, \mathbf{x}_{-i})$ in $\mathbf{x}_i$ for any fixed $\mathbf{x}_{-i}$ implies quasiconcavity). For examples, the games described in this article satisfy these conditions.

To overcome the problem of no equilibrium in pure strategies, one can restate the NE concept to contain mixed strategies [34], i.e., the possibility of choosing a randomization over a set of pure strategies. A mixed strategy NE of a strategic game is then defined as an NE of its mixed extension (see, e.g., [12] and [13] for details). An interesting result dealing with Nash equilibria in mixed strategy is that every strategic game with a finite strategy set has a mixed strategy NE [34], which in general does not hold for pure strategies.

Study of the uniqueness is much more involved and we refer the interested reader to the technical literature on the subject. However, it is important to recall here one of the simplest result dealing with uniqueness conditions, still based on the interpretation of the NE as a fixed-point solution. Roughly speaking, we can say that a pure strategy NE of $\mathcal{G}$, a fixed-point of the best-response function, is guaranteed to be unique if the best-response function (now assumed to be a one-to-one mapping) is a contraction in some vector norm [27], [35]. Contraction under a proper choice of norm, guarantees also convergence of (possibly) asynchronous algorithms based on the best-response function [35], as, e.g., the algorithms that will be introduced in this article.

Finally, it is important to remark that, even when the NE is unique, it needs not be Pareto efficient. A strategy profile $\mathbf{x} \in \mathcal{Q}$ is Pareto efficient (optimal) if there exists no other strategy $\mathbf{y} \in \mathcal{Q}$ that dominates $\mathbf{x}$, i.e., $u_i(\mathbf{y}) \geq u_i(\mathbf{x})$ for all $i \in \Omega$, and $u_j(\mathbf{y}) > u_j(\mathbf{x})$ for at least one $j \in \Omega$. This means that, given a strategic noncooperative game, there might exist proper coalitions among the players yielding an outcome of the game with the property that there is always (at least) one player who cannot profit by deviating by that action profile. In other words, an NE may be vulnerable to deviations by coalitions of players, even if it is not vulnerable to unilateral deviation by a single player. However, Pareto optimality in general comes at the price of a centralized optimization, which requires the full knowledge of the strategy sets and the payoff functions of all players. Such a centralized approach is not applicable in many practical applications in signal processing and communications, e.g., in emerging wireless networks, such as sensor networks, ad-hoc networks, CR systems, and pervasive computing systems. The NE solutions, instead, are more suitable to be computed using a decentralized approach that does require no exchange of information among the players. Different refinements of the NE concept have also been proposed in the literature to overcome some shortcomings of the NE solution (see, e.g., [36]–[37]).

VARIATIONAL INEQUALITIES PROBLEMS

The VI problem is as follows. Given a subset $\mathcal{K}$ of the Euclidean n-dimensional space $\mathbb{R}^n$ and a mapping $\mathbf{F}: \mathcal{K} \mapsto \mathbb{R}^n$, the VI problem, denoted $\text{VI}(\mathcal{K}, \mathbf{F})$, is to find a vector $\mathbf{x}^* \in \mathcal{K}$ (called a solution of the VI) such that

$$(\mathbf{x} - \mathbf{x}^*)^T \mathbf{F}(\mathbf{x}^*) \geq 0, \quad \forall \mathbf{x} \in \mathcal{K}. \tag{3}$$

Several interesting problems can be formulated as VI problems and some examples follow (see [27] for more source problems).

- *Solution of systems of equations.* The simplest example of VI is the problem of solving a system of equations. In fact, it is easy to see that if $\mathcal{K} = \mathbb{R}^n$, then $\text{VI}(\mathbb{R}^n, \mathbf{F})$ is equivalent to finding a $\mathbf{x}^* \in \mathbb{R}^n$ such that $\mathbf{F}(\mathbf{x}^*) = \mathbf{0}$. As special case, if the mapping $\mathbf{F}$ is affine, i.e., $\mathbf{F}(\mathbf{x}) = \mathbf{A}\mathbf{x} - \mathbf{b}$, the previous problem is equivalent to the classical system of equation $\mathbf{A}\mathbf{x}^* = \mathbf{b}$.
- *Fixed-point problems.* Given a mapping $\mathbf{T}: \mathcal{K} \mapsto \mathcal{K}$, the fixed-point problem is to find a vector $\mathbf{x}^* \in \mathcal{K}$ such that $\mathbf{T}(\mathbf{x}^*) = \mathbf{x}^*$. This problem can be converted into a VI format, simply by defining $\mathbf{F}(\mathbf{x}) = \mathbf{x} - \mathbf{T}(\mathbf{x})$.
- *Constrained and unconstrained optimization.* If $\mathcal{K}$ is convex and the mapping $\mathbf{F}$ in $\text{VI}(\mathcal{K}, \mathbf{F})$ is the gradient of a real-valued function $f: \mathcal{K} \mapsto \mathbb{R}$, then $\text{VI}(\mathcal{K}, \mathbf{F})$ represents a necessary conditions of optimality for the following optimization problem: find a point $\mathbf{x}^* \in \mathcal{K}$ such that $f(\mathbf{x}^*) \leq f(\mathbf{x})$, for all $\mathbf{x} \in \mathcal{K}$. Also, if the function f is convex, the reverse assertion is true, meaning that a point $\mathbf{x}^* \in \mathcal{K}$ minimizes f over $\mathcal{K}$ if and only if is a solution to $\text{VI}(\mathcal{K}, \nabla f)$, where ∇f denotes the gradient of f (the VI coincides with the first-order necessary and sufficient optimality conditions of a convex differentiable function). In particular, if we let $\mathcal{K} = \mathbb{R}^n$, we see that unconstrained convex optimization is also a VI problem.
- *Game theory problems.* Consider a strategic noncooperative game $\mathcal{G} = \langle \Omega, \mathcal{Q}, \mathbf{u} \rangle$ as defined in (1), and suppose that each $\mathcal{Q}_i \subset \mathbb{R}^{n_i}$ is convex and closed, and $u_i(\mathbf{x}_i, \mathbf{x}_{-i})$ is convex and continuously differentiable in $\mathbf{x}_i$, then a strategy profile $\mathbf{x}^*$ is an NE if and only if it solves the $\text{VI}(\mathcal{K}, \mathbf{F})$, with $\mathcal{K} = \mathcal{Q}_1 \times \cdots \times \mathcal{Q}_Q$ and $\mathbf{F}(\mathbf{x}) = -[\nabla_{\mathbf{x}_1}^T u_1(\mathbf{x}), \cdots, \nabla_{\mathbf{x}_Q}^T u_Q(\mathbf{x})]^T$, where $\nabla_{\mathbf{x}_i} u_i(\mathbf{x})$ denotes the gradient of $u_i(\mathbf{x})$ with respect to $\mathbf{x}_i$.

The theory and solution methods for various kinds of VIs are developed rather well and allow one to choose a suitable way to investigate each particular problem under consideration. For the purpose of this article, it is enough to recall some of the basic conditions for the existence of the solution to a VI, as they were used to obtain the results provided in the article for the proposed CR problems. A classical existence result reads as follows [26], [27]. The $\text{VI}(\mathcal{K}, \mathbf{F})$ is solvable if $\mathcal{K}$ is a nonempty, convex, and compact subset of a finite-dimensional Euclidean space; and $\mathbf{F}$ is a continuous mapping. The study of the uniqueness of the solution is much more involved and goes beyond the scope of this article. We refer the interested readers to [27, Ch. 2–3] for a detailed analysis of the topic. Many solution methods along with their convergence properties have been also proposed for VI in the literature; a detailed description can be found in [26] and [27].

CR SYSTEM MODEL

We consider a scenario composed of heterogeneous MIMO wireless systems (primary and secondary users) sharing the same physical resources, e.g., time, frequency, and space, as depicted in Figure 2. The setup may include MIMO peer-to-peer links, multiple access, or broadcast (single or multiantenna, flat, or frequency-selective) channels. The systems coexisting in the network do not cooperate with each other, and no centralized authority is assumed to handle the network access for the secondary users. Hence, it is natural to model the set of cognitive secondary users as vector interference channel, where the transmission over the generic qth MIMO channel with n_{T_q} transmit and n_{R_q} receive dimensions is given by the following baseband complex-valued signal model:

$$\mathbf{y}_q = \mathbf{H}_{qq}\mathbf{x}_q + \sum_{r \neq q} \mathbf{H}_{rq}\mathbf{x}_r + \mathbf{n}_q, \quad (4)$$

where the n_{T_q}-dimensional vector $\mathbf{x}_q$ is the signal transmitted by source q, $\mathbf{y}_q$ is the n_{R_q}-dimensional vector received by destination q, $\mathbf{H}_{qq}$ is the $n_{R_q} \times n_{T_q}$ channel matrix between the qth transmitter and the intended receiver, $\mathbf{H}_{rq}$ is the $n_{R_q} \times n_{T_r}$ cross-channel matrix between source r and destination q, and $\mathbf{n}_q$ is a zero-mean circularly symmetric complex Gaussian noise vector with arbitrary (nonsingular) covariance matrix $\mathbf{R}_{n_q}$, collecting the effect of both thermal noise and interference generated by the primary users. The first term on the right-hand side of (4) is the useful signal for link q, the second and third terms represent the multiuser interference (MUI) received by secondary user q and generated by the other secondary users and the primary users, respectively. The power constraint for each transmitter is

$$\mathcal{E}\{\|\mathbf{x}_q\|_2^2\} = \mathrm{Tr}(\mathbf{Q}_q) \leq \mathrm{P}_q, \quad (5)$$

where $\mathcal{E}\{\cdot\}$ denotes the expectation value and $\mathbf{Q}_q \succeq 0$ is the covariance matrix of the symbols transmitted by user q (the inequality "$\succeq 0$" stands for positive semidefinite), and P_q is the transmit power in units of energy per transmission.

The model in (4) represents a fairly general MIMO setup, describing multiuser transmissions over multiple channels, which may represent frequency channels (as in OFDM systems) [16]–[19], time slots [as in time division multiple access (TDMA) systems] [16]–[17], [19], or spatial channels (as in transmit/receive beamforming systems) [21].

Due to the distributed nature of the CR system, where there is neither a centralized control nor coordination among the secondary users, we focus on transmission techniques where no interference cancellation is performed and the MUI is treated as additive-colored noise at each receiver. Each channel is assumed to change sufficiently slow enough to be considered fixed during the whole transmission, so that the information theoretical results are meaningful. Moreover, perfect channel state information at both transmitter and receiver sides of each link is assumed. This includes the direct channel $\mathbf{H}_{qq}$ (but not the cross-channels $\{\mathbf{H}_{rq}\}_{r \neq q}$ from the other secondary users) as well as the covariance matrix of the noise plus MUI

$$\mathbf{R}_{-q}(\mathbf{Q}_{-q}) \triangleq \mathbf{R}_{n_q} + \sum_{r \neq q} \mathbf{H}_{rq}\mathbf{Q}_r\mathbf{H}_{rq}^H. \quad (6)$$

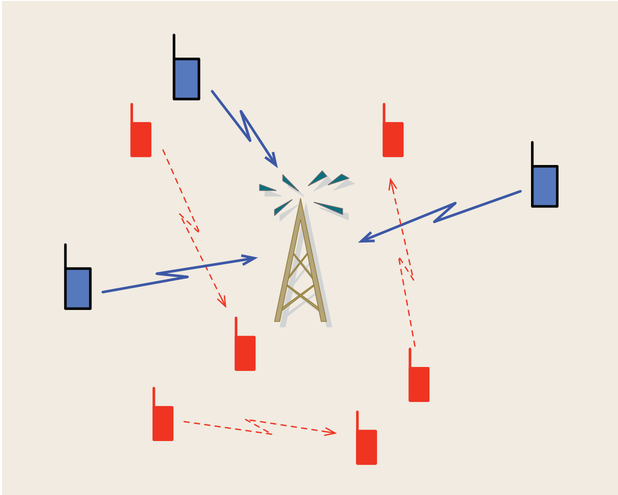
Within the assumptions made above, the maximum information rate on link q for a given set of user covariance matrices $\mathbf{Q}_1, \ldots, \mathbf{Q}_Q$, is [38]

$$R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) = \log\det(\mathbf{I} + \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q})\mathbf{H}_{qq}\mathbf{Q}_q), \quad (7)$$

where $\mathbf{Q}_{-q} \triangleq (\mathbf{Q}_r)_{r \neq q}$ is the set of all the users covariance matrices, except the qth one.

INTERFERENCE CONSTRAINTS: INDIVIDUAL AND CONSERVATIVE VERSUS GLOBAL AND FLEXIBLE

Different from traditional static or centralized spectrum assignment, opportunistic communications in CR systems enable secondary users to transmit with overlapping spectrum and/or coverage with primary users, provided that the degradation induced on the primary users' performance is null or tolerable [3], [4]. While the definition of degradation may be formulated mathematically in a number of ways, one common definition involves the imposition of some form of interference constraints on the secondary users, whose choice and implementation are a complex and open regulatory issue. Both deterministic and probabilistic interference constraints have been suggested in the literature [3], [4], namely: the maximum MUI interference power level perceived by any active primary user (the so-called temperature-interference limit) [1], [3] and the maximum probability that the MUI interference level at each primary user's receiver may exceed a prescribed threshold [4]. In this article, we will consider in detail deterministic interference constraints; in particular, we envisage the use of two classes of interference constraints, termed individual conservative constraints and global flexible constraints.



[FIG2] A conceptual illustration of a hierarchical CR system with primary users (uplink cellular system in blue) and secondary users (red pairs).

INDIVIDUAL CONSERVATIVE CONSTRAINTS

These constraints are defined individually for each secondary user (with the disadvantage that sometimes may result too conservative) to control the overall interference caused on the primary receivers. Specifically, we have the following:

- *Null-shaping constraints:*

$$\mathbf{U}_q^H \mathbf{Q}_q = \mathbf{0}, \quad (8)$$

where $\mathbf{U}_q$ is a tall matrix whose columns represent the spatial and/or the frequency "directions" along which user q is not allowed to transmit [$(\cdot)^H$ denotes the Hermitian of the matrix argument].

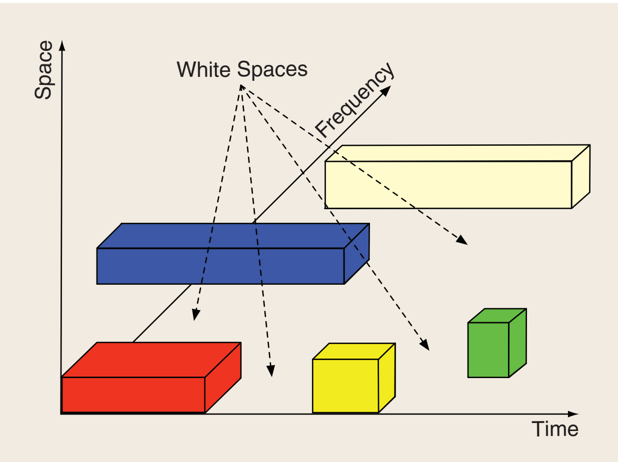
- *Soft- and peak-power-shaping constraints:*

$$\mathrm{Tr}(\mathbf{G}_q^H\mathbf{Q}_q\mathbf{G}_q) \leq P_{\mathrm{SU},q}^{\mathrm{ave}} \text{ and } \lambda_{\max}(\mathbf{G}_q^H\mathbf{Q}_q\mathbf{G}_q) \leq P_{\mathrm{SU},q}^{\mathrm{peak}}, \quad (9)$$

which represent a relaxed version of the null constraints with a constraint on the total average and peak average power radiated along the range space of matrix $\mathbf{G}_q$, where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue, and $P_{\mathrm{SU},q}^{\mathrm{ave}}$ and $P_{\mathrm{SU},q}^{\mathrm{peak}}$ are the maximum average and average peak power respectively that can be transmitted along the spatial and/or the frequency directions spanned by $\mathbf{G}_q$.

The null constraints are motivated in practice by the interference-avoiding paradigm in CR communications (also called white-space filling approach) [5], [39]: CR nodes sense the spatial, temporal, or spectral voids and adjust their transmission strategy to fill in the sensed white spaces, as illustrated in Figure 3. This white-space filling strategy is often considered to be the key motivation for the introduction and development of CR idea and has already been adopted as a core platform in emerging wireless access standards such as IEEE 802.22 Wireless Regional Area Networks (WRANs) [40]–[41]. One interesting note is that the FCC is in the second phase of testing white-space devices from a number of companies and research labs, meaning that the white-space filling paradigm is approaching. Observe that the structure of the null constraints in (8) is a very general form and includes, as particular cases, the imposition of nulls over: 1) frequency bands occupied by the primary users (the range space of $\mathbf{U}_q$ coincides with the subspace spanned by a set of IDFT vectors), 2) the time slots used by the primary users (the set of canonical vectors), and 3) angular directions identifying the primary receivers as observed from the secondary transmitters (the set of steering vectors representing the directions of the primary receivers as observed from the secondary transmitters). It is worth emphasizing that the use of the spatial domain can greatly improve the capabilities of cognitive users, as it allows them to transmit over the same frequency band and time slot without interfering.

The imposition of the null constraints at the secondary users' side requires an opportunity identification phase, through a proper sensing mechanism: Secondary users need to reliably detect weak primary signals of possibly different types over a targeted region and wide frequency band to identify white-space halls. In particular, the use of spatial null constraints requires the identification of the primary receivers, a task that is much more demanding than the detection of primary transmitters, if the primary receivers are passive devices, as in TV network or downlink



[FIG3] A conceptual illustration of resource utilization over time, frequency and space: the white-spaces are available for the transmissions of the secondary users.

cellular systems. Examples of solutions to this problem have recently been proposed in [4] and [42]–[43]. A recent overview of the challenges and possible solutions for the design of collaborative wideband sensing in CR networks can be found in [44[–[45]. The study of sensing in CR networks goes beyond the scope of this article. Hereafter, we thus assume perfect sensing from the secondary users.

While the white-space filling paradigm demands that cognitive transmissions be orthogonal (in space, time, or frequency) to primary transmissions, opportunistic communications involve simultaneous transmissions between primary and secondary users, provided that the required QoS of the primary users is preserved (also called interference-temperature controlled transmissions [3], [39], [43]). This can be done using the individual soft shaping constraints expressed in (9) that represent a constraint on the total average and peak average power allowed to be radiated (projected) along the directions spanned by the column space of matrix $\mathbf{G}_q$. For example, in a MIMO setup, the matrix $\mathbf{G}_q$ in (9) may contain, in its columns, the steering vectors identifying the directions of the primary receivers. By using these constraints, we assume that the power thresholds $P_{\mathrm{SU},q}^{\mathrm{ave}}$ and $P_{\mathrm{SU},q}^{\mathrm{peak}}$ at each secondary transmitter have been fixed in advance (imposed, e.g., by the network service provider, or legacy systems, or the spectrum body agency) so that the temperature-interference constraints at the primary receivers are met. For example, a possible (but conservative) choice for $P_{\mathrm{SU},q}^{\mathrm{ave}}$s and $P_{\mathrm{SU},q}^{\mathrm{peak}}$s is $P_{\mathrm{SU},q}^{\mathrm{ave}} = P_{\mathrm{PU}}^{\mathrm{ave}}/Q$ and $P_{\mathrm{SU},q}^{\mathrm{peak}} = P_{\mathrm{PU}}^{\mathrm{peak}}/Q$ for all q, where $P_{\mathrm{PU}}^{\mathrm{ave}}$ and $P_{\mathrm{PU}}^{\mathrm{peak}}$ are the overall maximum average and peak average interference tolerable by the primary user, and Q is the number of active secondary users. The assumption made above is motivated by all the practical CR scenarios where primary terminals are oblivious to the presence of secondary users, thus behaving as if no secondary activity was present (also called commons model).

Even though individual interference constraints (possibly in addition with the null constraints) will lead to totally distributed algorithms with no coordination between the primary and the secondary users, as we will show in the forthcoming sections, they sometimes may be too restrictive and thus marginalize the potential gains offered by the dynamic resource assignment mechanism. Since the temperature-interference limit [3] is given by the aggregate interference induced by all of the active secondary users to the primary users' receivers, it seems natural to limit instead such a aggregate interference, rather than the individual soft power and peak power constraints. This motivates the following global interference constraints.

GLOBAL FLEXIBLE CONSTRAINTS

These constraints, as opposed to the individual ones, are defined globally over all the secondary users

$$\sum_{q=1}^{Q}\mathrm{Tr}\left(\mathbf{G}_{q,p}^{H}\mathbf{Q}_q\mathbf{G}_{q,p}\right) \le P_{\mathrm{PU},p}^{\mathrm{ave}} \text{ and } \sum_{q=1}^{Q}\lambda_{\max}\left(\mathbf{G}_{q,p}^{H}\mathbf{Q}_q\mathbf{G}_{q,p}\right) \le P_{\mathrm{PU},p}^{\mathrm{peak}}, \tag{10}$$

where $P_{\mathrm{PU},p}^{\mathrm{ave}}$ and $P_{\mathrm{PU},p}^{\mathrm{peak}}$ are the maximum average and peak average interference tolerable by the pth primary user. As we will show in the forthcoming sections, these constraints in general lead to better performance of secondary users that imposing the conservative individual constraints. However, this gain comes at a price: The resulting algorithms require some signaling (albeit, very reduced) from the primary to the secondary users. They can be employed in all CR networks where an interaction between the primary and the secondary users is allowed, as, e.g., in the so-called property-right CR model (or spectrum leasing), where primary users own the spectral resource and possibly decide to lease part of it to secondary users in exchange for appropriate remuneration.

Within the CR context above, we formulate next the optimization problem for the transmission strategies of the secondary users under different combinations of power and individual/global interference constraints, and propose many iterative algorithms that converge to the solution along with their convergence properties.

NONCOOPERATIVE GAMES WITH CONSERVATIVE INDIVIDUAL CONSTRAINTS

We formulate the resource allocation problem among secondary users as a strategic noncooperative game, where the players are the secondary users and the payoff functions are the information rates on each link: Each secondary user q competes against the others by choosing the transmit covariance matrix $\mathbf{Q}_q$ (i.e., his strategy) that maximizes his own information rate $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ in (7), under different combination of power and individual interference constraints (the more complex global constraints are considered in the next section via the more general framework of VI). An NE of the game is reached when each user, given the strategy profiles of the others, does not get any rate increase by unilaterally changing his own strategy. For the sake of simplicity, we start by considering only power constraints (5) and individual null constraints (8), since they are suitable to model the white-space filling paradigm. Then, we consider more general opportunistic communications by allowing also soft-shaping interference constraints (9).

GAME WITH NULL CONSTRAINTS

Given the rate functions in (7), the rate maximization game among the secondary users in the presence of the power constraints (5) and null constraints (8) is formally defined as [46]

$$(\mathcal{G}_{\text{null}}): \begin{array}{ll} \underset{\mathbf{Q}_q \succeq 0}{\text{maximize}} & R_q(\mathbf{Q}_q, \mathbf{Q}_{-q}) \\ \text{subject to} & Tr(\mathbf{Q}_q) \leq P_q, \; \mathbf{U}_q^H \mathbf{Q}_q = \mathbf{0} \end{array} \quad (11)$$

for all $q \in \Omega$, where $\Omega \triangleq \{1, 2, \ldots, Q\}$ is the set of the players (the Q secondary users) and $R_q(\mathbf{Q}_q, \mathbf{Q}_{-q})$ is the payoff function of player q, defined in (7). Without the null constraints, the solution of each optimization problem in (11) would lead to the well-known MIMO waterfilling solution [38]. The presence of the null constraints modifies the problem and the solution for each user is not necessarily a waterfilling anymore. Nevertheless, we show now that introducing a proper projection matrix the solutions of (11) can still be efficiently computed via a waterfilling-like expression. To this end, we rewrite game $\mathcal{G}_{\text{null}}$ in a more convenient form as detailed next.

Observe that the null constraint $\mathbf{U}_q^H \mathbf{Q}_q = \mathbf{0}$ in (11) is equivalent to [46]

$$\mathbf{Q}_q = \mathbf{P}_q^{\perp} \mathbf{Q}_q \mathbf{P}_q^{\perp}, \quad (12)$$

where $\mathbf{P}_q^{\perp} = \mathbf{I} - \mathbf{U}_q \mathbf{U}_q^H$ denotes the orthogonal projection onto the orthogonal complement of the column space of $\mathbf{U}_q$. At this point, the problem can be further simplified by noting that the null constraint $\mathbf{Q}_q = \mathbf{P}_q^{\perp} \mathbf{Q}_q \mathbf{P}_q^{\perp}$ in (11) is redundant, provided that the original channels $\mathbf{H}_{rq}$ are replaced by the modified channels $\mathbf{H}_{rq}\mathbf{P}_r^{\perp}$. Then, the final formulation becomes

$$\begin{array}{ll} \underset{\mathbf{Q}_q \succeq 0}{\text{maximize}} & \log\det(\mathbf{I} + \mathbf{P}_q^{\perp} \mathbf{H}_{qq}^H \widetilde{\mathbf{R}}_{-q}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq} \mathbf{P}_q^{\perp} \mathbf{Q}_q) \\ \text{subject to} & \text{Tr}(\mathbf{Q}_q) \leq P_q \end{array} \quad (13)$$

for all $q \in \Omega$, where

$$\widetilde{\mathbf{R}}_{-q}(\mathbf{Q}_{-q}) \triangleq \mathbf{R}_{n_q} + \sum_{r \neq q} \mathbf{H}_{rq} \mathbf{P}_r^{\perp} \mathbf{Q}_r \mathbf{P}_r^{\perp} \mathbf{H}_{rq}^H. \quad (14)$$

This is due to the fact that, for any user q, any optimal solution $\mathbf{Q}_q^*$ in (13), the MIMO waterfilling solution, will be orthogonal to the null space of $\mathbf{H}_{qq}\mathbf{P}_q^{\perp}$, whatever $\widetilde{\mathbf{R}}_{-q}(\mathbf{Q}_{-q}^*)$ is (recall that $\widetilde{\mathbf{R}}_{-q}(\mathbf{Q}_{-q})$ is positive definite for all feasible $\mathbf{Q}_{-q}$), implying that the range space of each $\mathbf{Q}_q^*$ is contained in or equal to that of $\mathbf{U}_q^{\perp}$. Note that (13) can be further simplified by using $\mathbf{R}_{-q}(\mathbf{Q}_{-q})$ rather than $\widetilde{\mathbf{R}}_{-q}(\mathbf{Q}_{-q})$ [46].

Building on the equivalence of (11) and (13), we can focus on the game in (13) and apply the framework proposed in [23] to fully characterize the game $\mathcal{G}_{\text{null}}$, by deriving the structure of the Nash equilibria and the conditions guaranteeing both the existence/uniqueness of the equilibrium and the global convergence of the proposed distributed algorithms.

NASH EQUILIBRIA EXISTENCE AND UNIQUENESS

To write the Nash equilibria of game $\mathcal{G}_{\text{null}}$ in a convenient form, we introduce first the MIMO waterfilling operator defined, for any $q \in \Omega$, as

$$\text{WF}_q(\mathbf{X}) \triangleq \mathbf{U}_X(\mu_{q,X}\mathbf{I} - \mathbf{D}_X^{-1})^+ \mathbf{U}_X^H, \quad (15)$$

where $\mathbf{U}_X$ is the (semi) unitary matrix of the eigenvectors associated to the positive eigenvalues of $\mathbf{X}$, $\mathbf{D}_X$ is the diagonal matrix with the positive eigenvalues, $\mu_{q,X} > 0$ is the water level chosen to satisfy the power constraint $\text{Tr}\{(\mu_{q,X}\mathbf{I} - \mathbf{D}_X^{-1})^+\} = P_q$, with $(x)^+ \triangleq \max(0, x)$ applied component-wise, and $\mathbf{I}$ is the identity matrix. The solution to the single user optimization problem in (11), the best response of player q is [46]

$$\mathbf{Q}_q^* = \text{T}_q(\mathbf{Q}_{-q}) \triangleq \mathbf{U}_q^{\perp} \text{WF}_q(\mathbf{U}_q^{\perp H} \mathbf{H}_{qq}^H \mathbf{R}_{-q}^{-1}(\mathbf{Q}_{-q}) \mathbf{H}_{qq} \mathbf{U}_q^{\perp}) \mathbf{U}_q^{\perp H}, \quad (16)$$

where $\mathbf{U}_q^{\perp}$ is the semiunitary matrix orthogonal to $\mathbf{U}_q$ (note that $\mathbf{P}_q^{\perp} = \mathbf{U}_q^{\perp}\mathbf{U}_q^{\perp H}$), and $\text{WF}_q(\cdot)$ and $\mathbf{R}_{-q}(\mathbf{Q}_{-q})$ are defined in (15) and (6), respectively. Using (16), we can now characterize the Nash equilibria of the game $\mathcal{G}_{\text{null}}$ in a compact way as the solutions to the following waterfilling fixed-point equation:

$$\mathbf{Q}_q^* = \text{T}_q(\mathbf{Q}_{-q}^*), \quad \forall q \in \Omega, \quad (17)$$

which always admits a solution, for any set of channel matrices, power constraints of the users, and null-shaping constraints (the game $\mathcal{G}_{\text{null}}$ indeed satisfies basic existence results, as given in the previous section).

The structure of the best response as given in (16) (and thus the Nash equilibria) shows that the null constraints in the transmissions of secondary users can be handled without affecting the computational complexity: The optimal transmission strategy for each user q can be efficiently computed via a MIMO waterfilling solution, provided that the original channel matrix $\mathbf{H}_{qq}$ is replaced by $\mathbf{H}_{qq}\mathbf{U}_q^{\perp}$. This result has an intuitive interpretation: To guarantee that each user q does not transmit over a given subspace (spanned by the columns of $\mathbf{U}_q$), whatever the strategies of the other users are, while maximizing his information rate, it is enough to induce in the original channel matrix $\mathbf{H}_{qq}$ a null space that (at least) coincides with the subspace where the transmission is not allowed. This is precisely what is done in the payoff functions in (13) by replacing $\mathbf{H}_{qq}$ with $\mathbf{H}_{qq}\mathbf{P}_q^{\perp}$.

The waterfilling-like structure of the Nash equilibria in (17) is also instrumental to study the uniqueness of the equilibrium as well as the convergence of distributed algorithms. In fact, invoking the interpretation of the MIMO waterfilling solution as a matrix projection over a proper convex set as described in [21] and [23], one can obtain sufficient conditions guaranteeing the uniqueness of the NE of $\mathcal{G}_{\text{null}}$ by studying the contraction properties of the waterfilling projection. For example, the NE of $\mathcal{G}_{\text{null}}$ is unique if one of the two following set of conditions is satisfied (more general but less intuitive conditions are given in [46]):

Low-received MUI:

$$\sum_{r \neq q} \text{innr}_q \cdot \rho(\mathbf{H}_{rq}^H \mathbf{H}_{rq}) \rho(\mathbf{H}_{qq}^{\#H} \mathbf{H}_{qq}^{\#}) < 1, \ \forall q \in \Omega, \qquad \text{(C1)}$$

Low-generated MUI:

$$\sum_{q \neq r} \text{innr}_q \cdot \rho(\mathbf{H}_{rq}^H \mathbf{H}_{rq}) \rho(\mathbf{H}_{qq}^{\#H} \mathbf{H}_{qq}^{\#}) < 1, \ \forall r \in \Omega, \qquad \text{(C2)}$$

where $(\cdot)^{\#}$ denotes the Moore-Penrose pseudoinverse [47], and innr_q is the interference-plus-noise to noise ratio, defined as

$$\text{innr}_q \triangleq \frac{\rho\left(\mathbf{R}_{n_q} + \sum_{r \neq q} P_r \mathbf{H}_{rq} \mathbf{H}_{rq}^H\right)}{\lambda_{\min}(\mathbf{R}_{n_q})}, \qquad (18)$$

with $\rho(\mathbf{A})$ denoting the spectral radius of $\mathbf{A}$ (i.e., the maximum of the modulus of the eigenvalues).

Conditions (C1) and (C2) have an interesting interpretation: The uniqueness of the NE is ensured if the interference among secondary users is sufficiently small. Condition (C1) can be indeed interpreted as a constraint on the maximum amount of interference that each receiver can tolerate, whereas (C2) introduces an upper bound on the maximum level of interference that each transmitter of the secondary users is allowed to generate.

DISTRIBUTED ALGORITHMS

We focus now on distributed algorithms that converge to the NE of game $\mathcal{G}_{\text{null}}$. We consider totally asynchronous distributed algorithms, meaning that in the updating procedure some users are allowed to change their strategy more frequently than the others, and they might even perform these updates using outdated information on the interference caused by the others. To provide a formal description of the proposed asynchronous MIMO IWFA, we briefly recall some intermediate definitions, as given in [21].

We assume, without loss of generality (w.l.o.g.), that the set of times at which one or more users update their strategies is the discrete set $\mathcal{T} = \mathbb{N}_+ = \{0, 1, 2, \ldots\}$. Let $\mathbf{Q}_q^{(n)}$ denote the covariance matrix of the vector signal transmitted by user q at the nth iteration, and let $\mathcal{T}_q \subseteq \mathcal{T}$ denote the set of times n at which $\mathbf{Q}_q^{(n)}$ is updated (thus, at time $n \notin \mathcal{T}_q$, $\mathbf{Q}_q^{(n)}$ is left unchanged). Let $\tau_r^q(n)$ denote the most recent time at which the interference from user r is perceived by user q at the nth iteration (observe that $\tau_r^q(n)$ satisfies $0 \leq \tau_r^q(n) \leq n$). Hence, if user q updates his own covariance matrix at the nth iteration, then he chooses his optimal $\mathbf{Q}_q^{(n)}$, according to his best-response $\mathrm{T}_q(\mathbf{Q}_{-q})$ defined in (16) and using the interference level caused by

$$\mathbf{Q}_{-q}^{(\tau^q(n))} \triangleq \left(\mathbf{Q}_1^{(\tau_1^q(n))}, \ldots, \mathbf{Q}_{q-1}^{(\tau_{q-1}^q(n))}, \mathbf{Q}_{q+1}^{(\tau_{q+1}^q(n))}, \ldots, \mathbf{Q}_Q^{(\tau_Q^q(n))}\right). \quad (19)$$

Some standard conditions in asynchronous convergence theory that are fulfilled in any practical implementation need to be satisfied by the schedule $\{\tau_r^q(n)\}$ and $\{\mathcal{T}_q\}$; we refer to [18] and [21] for the details.

Using the above notation, the asynchronous MIMO IWFA is formally described in Algorithm 1, where the best response $\mathrm{T}_q(\cdot)$ of each user is defined in (16). Interestingly, as $n \rightarrow +\infty$, the algorithm converges to the solution of game under the same conditions guaranteeing the uniqueness of the NE [e.g., (C1) or (C2)], for any set of feasible initial conditions, and updating schedule [23], [46].

ALGORITHM 1: MIMO ASYNCHRONOUS IWFA

`Data` : any feasible $\mathbf{Q}_q^{(0)}$ for all $q = 1, \ldots, Q$.

1: Set $n = 0$;

2: `repeat`

3: $$\mathbf{Q}_q^{(n+1)} = \begin{cases} \mathrm{T}_q\left(\mathbf{Q}_{-q}^{(\tau^q(n))}\right), & \text{if } n \in \mathcal{T}_q, \\ \mathbf{Q}_q^{(n)}, & \text{otherwise;} \end{cases} \ \forall q \in \Omega \qquad (20)$$

4: `until the prescribed convergence criterion is satisfied.`

Algorithm 1 contains as special cases a plethora of algorithms, each one obtained by a possible choice of the scheduling of the users in the updating procedure (i.e., the parameters $\{\tau_r^q(n)\}$ and $\{\mathcal{T}_q\}$). Two well-known special cases are the sequential and the simultaneous MIMO IWFA, where the users update their own strategies sequentially and simultaneously, respectively. Interestingly, since conditions (C1)–(C2) do not depend on the particular choice of $\{\mathcal{T}_q\}$ and $\{\tau_r^q(n)\}$, the important result coming from the convergence analysis is that all the algorithms resulting as special cases of the asynchronous MIMO IWFA are guaranteed to reach the unique NE of the game, under the same set of convergence conditions. Moreover they have the following desired properties:

- *Low complexity and distributed nature:* Even in the presence of null constraints, the best response $T_q(\cdot)$ of each user q can be efficiently and locally computed using a MIMO waterfilling-based solution, provided that each channel H_{qq} is replaced by the channel $H_{qq}U_q^{\perp}$. Thus, Algorithm 1 can be implemented in a distributed way, since each user only needs to measure the overall interference-plus-noise covariance matrix $R_{-q}(Q_{-q})$ and waterfill over $U_q^{\perp H}H_{qq}^H R_{-q}^{-1}(Q_{-q})H_{qq}U_q^{\perp}$.
- *Robustness:* Algorithm 1 is robust against missing or outdated updates of secondary users. This feature strongly relaxes the constraints on the synchronization of the secondary users' updates with respect to those imposed, for example, by the simultaneous or sequential updating schemes.
- *Fast convergence behavior:* The simultaneous version of the proposed algorithm converges in a very few iterations, even in networks with many active secondary users. As expected, the sequential IWFA is slower than the simultaneous IWFA, especially if the number of active secondary users is large, since each user is forced to wait for all the users scheduled in advance, before updating his own covariance matrix. This intuition is formalized in [17] and [18], where the authors provided the expression of the asymptotic convergent factor of both the sequential and simultaneous IWFAs.
- *Control of the radiated interference:* Thanks to the game theoretical formulation including null constraints, the proposed asynchronous IWFA does not suffer of the main drawback of the classical IWFA-based algorithms 14], [21] (i.e., the violation of the temperature-interference limit [3]).

GAME WITH NULL CONSTRAINTS VIA VIRTUAL NOISE SHAPING

We have seen how to deal efficiently with null constraints in the rate maximization game. Under conditions (C1)–(C2), the NE is unique and Algorithm 1 asymptotically converges to this solution. However these conditions depend, among all, on the interference generated by the primary users (through the innr_qs), and thus they may not be satisfied for some interference profile, which is an undesired result. In such a case, the NE might not be unique and there is no guarantee that the proposed algorithms converge to an equilibrium. To overcome this issue, we propose here an alternative approach to impose null constraints (8) on the transmissions of secondary users based on the introduction of virtual interferers. This leads to a new game with more relaxed uniqueness and convergence conditions. The solutions of this new game are in general different to the Nash equilibria of $\mathcal{G}_{\text{null}}$, but the two games are numerically shown to have almost the same performance in terms of sum-rate.

The idea behind this alternative approach can be easily understood if one considers the transmission over SISO frequency-selective channels, where all the channel matrices have the same eigenvectors (the DFT vectors): to avoid the use of a given subchannel, it is sufficient to introduce a "virtual" noise with sufficiently high power over that subchannel. The same idea cannot be directly applied to the MIMO case, as arbitrary MIMO channel matrices have different right/left singular vectors from each other. Nevertheless, we show how to bypass this difficulty to design the covariance matrix of the virtual noise (to be added to the noise covariance matrix of each secondary receiver), so that all the Nash equilibria of the game satisfy the null constraints along the specified directions. For the sake of simplicity, we assume here nonsingular square channel matrices H_{qq}, for all $q \in \Omega$. Let us consider the following strategic noncooperative game [46]:

$$(\mathcal{G}_\alpha): \quad \begin{array}{ll} \underset{Q_q \succeq 0}{\text{maximize}} & \log\det(I + H_{qq}^H R_{-q,\alpha}^{-1}(Q_{-q})H_{qq}Q_q) \\ \text{subject to} & \operatorname{Tr}(Q_q) \le P_q \end{array} \quad \forall q \in \Omega \tag{21}$$

where

$$\begin{aligned} R_{-q,\alpha}(Q_{-q}) &\triangleq R_{-q}(Q_{-q}) + \alpha \hat{U}_q \hat{U}_q^H \\ &= R_{n_q} + \sum_{r \ne q} H_{rq}Q_r H_{rq}^H + \alpha \hat{U}_q \hat{U}_q^H \end{aligned} \tag{22}$$

denotes the MUI-plus-noise covariance matrix observed by secondary user q, plus the covariance matrix $\alpha\hat{U}_q\hat{U}_q^H$ of the virtual interference along the range space of the $\hat{U}_q$, where $\hat{U}_q$ is a (strictly) tall matrix, and α is a positive constant. Our interest is on deriving the asymptotic properties of the solutions of $\mathcal{G}_\alpha$, as $\alpha \to +\infty$, and the structure of $\hat{U}_q$'s making the null constraints in (8) satisfied.

NASH EQUILIBRIA: EXISTENCE AND UNIQUENESS

Game $\mathcal{G}_\alpha$ always admits an NE, for any set of channel matrices, transmit power of the users, virtual interference matrices $\hat{U}_q$'s, and $\alpha > 0$ (the game indeed satisfies basic existence results as given so far). Moreover, it can be proved that the solution to (21) is unique irrespective of the value of the $\hat{U}_q$s and $\alpha > 0$ if, e.g., one of the following two conditions are satisfied (more general conditions are given in [46]):

$$\text{Low-received MUI:} \quad \sum_{r \ne q} \rho(H_{rq}^H H_{qq}^{\#H} H_{qq}^{\#} H_{rq}) < 1, \ \forall q \in \Omega, \tag{C3}$$

$$\text{Low-generated MUI:} \quad \sum_{q \ne r} \rho(H_{rq}^H H_{qq}^{\#H} H_{qq}^{\#} H_{rq}) < 1, \ \forall r \in \Omega. \tag{C4}$$

Observe that, since conditions (C3)–(C4) do not depend on the transmission strategies of the primary users, game $\mathcal{G}_\alpha$ has the desired property that the uniqueness of the NE is robust against the interference generated by the primary users. Moreover, under (C3)–(C4), it can be shown that, asymptotically for $\alpha \to +\infty$, the unique solution to (21) indeed satisfies the null constraints (8), provided that virtual interferer's matrices are chosen so that $\hat{U}_q = H_{qq}U_q$ [46]; which provides an alternative way to impose the null constraints in (8) with respect to $\mathcal{G}_{\text{null}}$. We refer in the following to the game $\mathcal{G}_\alpha$ when $\alpha \to +\infty$ and each $\hat{U}_q = H_{qq}U_q$ as $\mathcal{G}_\infty$, and denote by Q_∞^* an NE of $\mathcal{G}_\infty$. It is worth emphasizing that, even though each Q_∞^* satisfies the null constraints (8), game $\mathcal{G}_\infty$ is a rather artificial formulation that does not correspond any physical scenario (in the game $\mathcal{G}_\infty$, the best-response strategy of each player is a waterfilling but over the fictitious channel $\hat{U}_q^{\perp H}H_{qq}$ rather than on the physical channel $\mathbf{H}_{qq}$ [46]). Nevertheless, we will show that the performance of $\mathcal{G}_\infty$ and $\mathcal{G}_{\text{null}}$ are (almost) the same in terms of sum-rate achievable at the NE.

DISTRIBUTED ALGORITHMS

To reach the Nash equilibria of game $\mathcal{G}_\alpha$ while satisfying the null constraints (for sufficiently large α), one can use the asynchronous IWFA as given in Algorithm 1, where the best-response $\mathrm{T}_q(\mathbf{Q}_{-q})$ in (20) is replaced by the following:

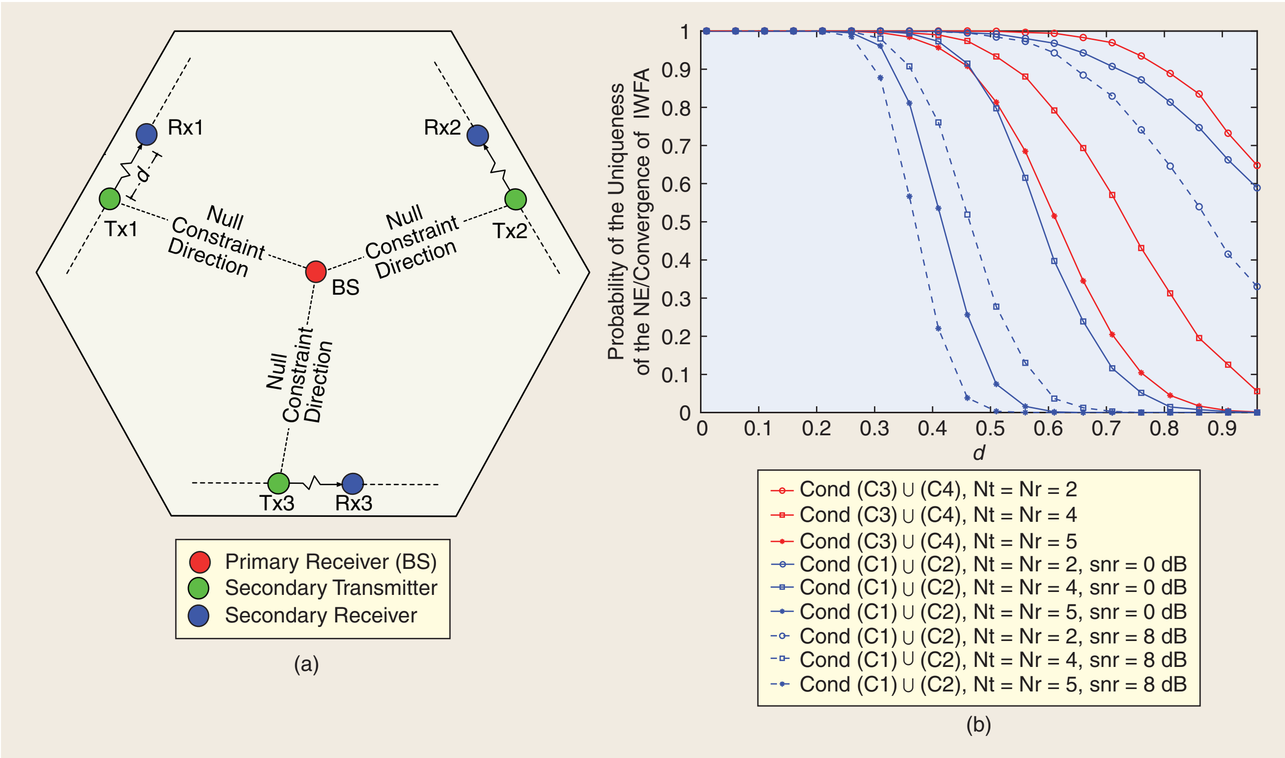
$$\mathrm{T}_{q,\alpha}(\mathbf{Q}_{-q}) \triangleq \mathrm{WF}_q(\mathbf{H}_{qq}^H \mathbf{R}_{-q,\alpha}^{-1}(\mathbf{Q}_{-q,})\mathbf{H}_{qq}), \qquad (23)$$

where the MIMO waterfilling operator WF_q is defined in (15), and the null constraints $\hat{\mathbf{U}}_q$ in $\mathbf{R}_{-q,\alpha}$ are $\hat{\mathbf{U}}_q = \mathbf{H}_{qq}\mathbf{U}_q$. Observe that such an algorithm has the same nice properties of the algorithm proposed to reach the Nash equilibria of the game $\mathcal{G}_{\text{null}}$. In particular, the best-response of each player q can be still efficiently and locally computed via a MIMO waterfilling-like solution, provided that the virtual interference covariance matrix $\alpha\mathbf{U}_q\mathbf{U}_q^H$ is added to the MUI covariance matrix $\mathbf{R}_{-q}(\mathbf{Q}_{-q})$ measured at the qth receiver. The global convergence of the algorithm as $n \to \infty$ is guaranteed under conditions (C3) or (C4) [46].

COMPARISON OF UNIQUENESS/CONVERGENCE CONDITIONS

Since the uniqueness/convergence conditions given so far depend on the channel matrices $\{\mathbf{H}_{rq}\}_{r,q\in\Omega}$, there is a nonzero probability that they will not be satisfied for a given channel realization drawn from a given probability space. To quantify the adequacy of our conditions, we tested them over a set of random channel matrices whose elements are generated as circularly symmetric complex Gaussian random variables with variance equal to the inverse of the square distance between the associated transmitter-receiver links (flat-fading channel model). We consider a hierarchical CR network as depicted in Figure 4(a), composed of three secondary user MIMO links and one primary user [the base station (BS)], sharing the same band. To preserve the QoS of the primary users, null constraints are imposed on the secondary users in the direction of the receiver of the primary user. In Figure 4(b), we plot the probability that conditions (C1) or (C2) and (C3) or (C4) are satisfied versus the intra-pair distance $d \in (0;1)$ (normalized by the cell's side) [see Figure 4(a)] between each secondary transmitter and the corresponding receiver (assumed for the simplicity of representation to be equal for all the secondary links), for different values of the transmit/receive antennas. Since conditions (C1)–(C2) depend on the interference generated by the primary user and the power budgets of the secondary users, we considered two different values of the SNR at the receivers of the secondary users, namely $\text{snr}_q \triangleq P_q/\sigma_{q,\text{tot}}^2 = 0$ dB and $\text{snr}_q = 8$ dB, for all $q \in \Omega$, where $\sigma_{q,\text{tot}}^2$ is the variance of thermal noise plus the interference generated by the primary user over all the substreams.

As expected, the probability of the uniqueness of the NE of both games $\mathcal{G}_{\text{null}}$ and $\mathcal{G}_\alpha$ and convergence of the IWFAs increases as each secondary transmitter approaches his receiver, corresponding to a decrease of the overall MUI. Moreover, conditions (C1)–(C2) are confirmed to be stronger than (C3)–(C4) whatever the number of transmit/receive



[FIG4] (a) CR MIMO system. (b) Probability of the uniqueness of the NE of games $\mathcal{G}_{\text{null}}$ and $\mathcal{G}_\alpha$ and convergence of the asynchronous IWFA as a function of the normalized intra-pair distance $d \in (0, 1)$, for the CR MIMO system given in (a).

antennas, the intra-pair distance d, and the SNR value are, implying that game $\mathcal{G}_\alpha$ admits milder uniqueness/convergence conditions than those of the original game $\mathcal{G}_{null}$.

PERFORMANCE OF $\mathcal{G}_{null}$ AND $\mathcal{G}_\infty$

As an example, in Figure 5, we compare games $\mathcal{G}_{null}$ and $\mathcal{G}_\infty$ in terms of sum-rate. Specifically, in Figure 5(a), we plot the average sum-rate at the (unique) NE of the games $\mathcal{G}_{null}$ and $\mathcal{G}_\infty$ for the CR network depicted in Figure 4(a) as a function of the intra-pair distance $d \in (0, 1)$ among the links, for different numbers of transmit/receive antennas. In Figure 5(b), we plot the outage sum-rate values, for the same systems as in Figure 5(a) and $d = 0.5$. For each secondary user, a null constraint in the direction of the receiver of the primary user is imposed. From the figures one infers that games $\mathcal{G}_{null}$ and $\mathcal{G}_\infty$ have almost the same performance in terms of sum-rate at the NE; even if in the game $\mathcal{G}_\infty$, given the strategies of the others, each player does not maximize his own rate, as happens in the game $\mathcal{G}_{null}$. This is due to the fact that the Nash equilibria of game $\mathcal{G}_{null}$ are in general not Pareto efficient. The above results indicate then that game $\mathcal{G}_\alpha$, with sufficiently large α, may be a valid alternative to game $\mathcal{G}_{null}$ to impose the null constraints with more relaxed conditions for convergence.

GAME WITH SOFT AND NULL CONSTRAINTS

We focus now on the rate maximization in the presence of both individual null and soft shaping constraints. The resulting game can be formulated as follows:

$$(\mathcal{G}_{soft}): \quad \begin{array}{ll} \underset{Q_q \succeq 0}{\text{maximize}} & R_q(Q_q, Q_{-q}) \\ \text{subject to} & Tr(G_q^H Q_q G_q) \le P_{SU,q}^{ave} \\ & \lambda_{max}(G_q^H Q_q G_q) \le P_{SU,q}^{peak} \\ & U_q^H Q_q = 0 \end{array} \qquad (24)$$

for all $q \in \Omega$, where the transmit power constraint (5) has been absorbed into the trace soft constraint for convenience [in practice, the transmit power constraint in (24) will be dominated by the trace shaping constraint, which motivates the absence in (24) of an explicit power constraint as given in (5)].

NASH EQUILIBRIA: EXISTENCE AND UNIQUENESS

We can derive the properties of the Nash equilibria of $\mathcal{G}_{soft}$ similarly to what we did for game $\mathcal{G}_{null}$. For any $q \in \Omega$, define the tall matrix $\overline{U}_q$ as $\overline{U}_q \triangleq G_q^{\#} U_q$, introduce the semiunitary matrix $\overline{U}_q^{\perp}$ orthogonal to $\overline{U}_q$, and the set of modified channels $\overline{H}_{rq}$, defined as

$$\overline{H}_{rq} = H_{rq} G_r^{\#H} \overline{U}_r^{\perp}, \ \forall r, q \in \Omega. \qquad (25)$$

Finally, to write the Nash equilibria in a convenient form, we introduce the modified waterfilling operator $\overline{WF}_q$, defined as

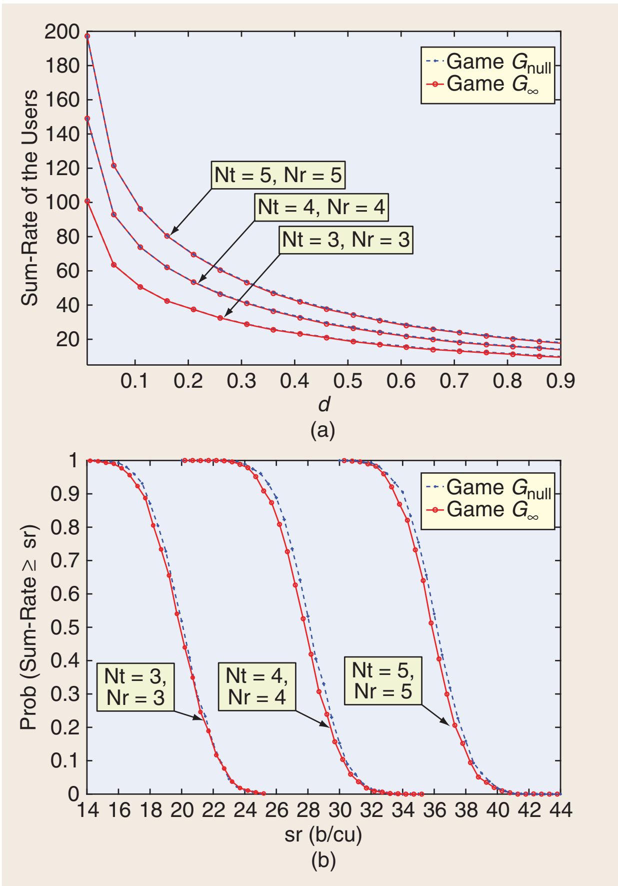
$$\overline{WF}_q(X) \triangleq U_X[\mu_{q,X} I - D_X^{-1}]_0^{P_q^{peak}} U_X^H, \qquad (26)$$

where U_X and D_X are defined as in (15), $[x]_a^b$ is the projection onto $[a, b]$ (i.e., $[x]_a^b = a$ if $x \le a$, $[x]_a^b = x$ if $a \le x \le b$, and $[x]_a^b = b$ if $x \ge b$) and is applied component-wise, and $\mu_{q,X} > 0$ is the water-level chosen to satisfy either the trace or the peak power constraint with equality [see, e.g., [48] for practical algorithms to compute the waterlevel $\mu_{q,X}$ in (26)]. Using the above definitions, we can write the best response of each player q, given $Q_{-q} \succeq 0$, as [46]

$$\overline{T}_q(Q_{-q}) \triangleq G_q^{\#^H} \overline{U}_q^{\perp} \overline{WF}_q(\overline{H}_{qq}^H R_{-q}^{-1}(Q_{-q}) \overline{H}_{qq}) \overline{U}_q^{\perp H} G_q^{\#}, \qquad (27)$$

showing that the optimal transmission strategy of each user leads to a diagonalizing transmission with a proper power allocation, after pre/postmultiplication by matrix $G_q^{\#H} \overline{U}_q^{\perp}$. Thus, even in the presence of soft constraints, the optimal transmission strategy of each user q, given the strategies Q_{-q} of the others, can be efficiently computed via a MIMO waterfilling-like solution. Note that the best response in (27) satisfy the null constrains, since $U_q^H G_q^{\#^H} \overline{U}_q^{\perp} = 0$ and thus $U_q^H Q_q^* = 0$ for all q.

The Nash equilibria of $\mathcal{G}_{soft}$ can be written as a fixed-point equation of the best response mapping $\overline{T}_q(Q)$ like (17), which always exist, for any set of channel matrices and null/soft shaping constraints. As far as the uniqueness of the NE is concerned, similar sufficient conditions to (C1)–(C2) can be obtained: it is sufficient to replace the original channel matrices $\tilde{H}_{rq}$ with the modified channels $\overline{H}_{rq}$ defined in (25) [46].



[FIG5] Performance of games $\mathcal{G}_{null}$ and $\mathcal{G}_\infty$ in terms of Nash equilibria for the CR MIMO system given in Figure 4. (a) Average sum-rate at the NE versus the normalized intra-pair distance $d \in (0, 1)$ for $d = 0.5$; (b) Outage sum-rate for both games $\mathcal{G}_{null}$ (plus-mark dashed-dot blue line curves) and $\mathcal{G}_\infty$ (circle-mark solid red line curves).

DISTRIBUTED ALGORITHMS

Similarly to games $\mathcal{G}_{\text{null}}$ and $\mathcal{G}_{\alpha}$, the Nash equilibria of game $\mathcal{G}_{\text{soft}}$ can be reached using the asynchronous IWFA algorithm given in Algorithm 1, based on the mapping $\overline{\text{T}}_q(\mathbf{Q}_{-q})$ defined in (27). Observe that such an algorithm has the same nice properties of the algorithm proposed to reach the Nash equilibria of game $\mathcal{G}_{\text{null}}$, such as low complexity, distributed and asynchronous nature, and fast convergence behavior. Moreover, thanks to the game theoretical formulation including null and/or soft-shaping constraints, the algorithm does not suffer of the main drawback of the classical sequential IWFA [14], [24], [25], i.e., the violation of the temperature-interference limits [3]. As for games $\mathcal{G}_{\text{null}}$ and $\mathcal{G}_{\alpha}$, under conditions guaranteeing the uniqueness of the NE, the asynchronous MIMO IWFA based on the mapping in (27), asymptotically converges to the NE, for any set of feasible initial conditions, and updating schedule [46].

COMPETITIVE RESOURCE SHARING BASED ON VI WITH GLOBAL FLEXIBLE INTERFERENCE CONSTRAINTS

We consider now the design of CR system in (4), including the global interference constraints in (10), instead of the conservative individual constraints as in the previous section. For the sake of simplicity, we focus here only on block transmissions over SISO frequency-selective channels. It is well known that, in such a case, multicarrier transmission is capacity achieving for large block-length [38]. This allows the simplification of the system model in (4), since each channel matrix $\mathbf{H}_{rq}$ becomes a $N \times N$ Toeplitz circulant matrix with eigendecomposition $\mathbf{H}_{rq} = \mathbf{F}\mathbf{D}_{rq}\mathbf{F}^H$, where $\mathbf{F}$ is the normalized IFFT matrix, i.e., $[\mathbf{F}]_{ij} \triangleq e^{j2\pi(i-1)(j-1)/N}/\sqrt{N}$ for $i,j = 1,\ldots,N$, N is the length of transmitted block, $\mathbf{D}_{rq} = \text{diag}(\{H_{rq}(k)\}_{k=1}^{N})$ is the diagonal matrix whose kth diagonal entry is the frequency-response of the channel between source r and destination q at carrier k, and $\mathbf{R}_{n_q} = \text{diag}(\{\sigma_q^2(k)\}_{k=1}^{N})$.

Under this setup, the strategy of each secondary user q becomes the power allocation $\mathbf{p}_q = \{p_q(k)\}_{k=1}^{N}$ over the N carriers and the payoff function in (7) reduces to the information rate over the N parallel channels

$$r_q(\mathbf{p}_q, \mathbf{p}_{-q}) = \sum_{k=1}^{N} \log\left(1 + \frac{|H_{qq}(k)|^2 p_q(k)}{\sigma_q^2(k) + \sum_{r \neq q} |H_{rq}(k)|^2 p_r(k)}\right). \quad (28)$$

Local power constraints and global interference constraints are imposed on the secondary users. The admissible strategy set of each player q associated to local power constraints is then

$$\mathcal{P}_q \triangleq \left\{\mathbf{p} : \sum_{k=1}^{N} p(k) \leq P_q, \quad 0 \leq \mathbf{p} \leq \mathbf{p}_q^{\max}\right\}, \quad (29)$$

where we also included possibly (local) spectral mask constraints $\mathbf{p}_q^{\max} = (p_q^{\max}(k))_{k=1}^{N}$ [the vector inequality in (29) has to be intended component-wise]. The global interference constraints in (10) impose a upper bound on the value of the per-carrier and total interference (the temperature-interference limit [3]) that can be tolerated by each primary user. For the sake of simplicity, here, we focus only on per-carrier interference constraints imposed by each primary user $p = 1, \ldots, P$ (both per-carrier and total interference constraints are considered in [49]):

$$\sum_{q=1}^{Q} |G_{qp}(k)|^2 p_q(k) \leq P_{p,k}^{\text{peak}}, \; \forall k = 1, \ldots, N, \quad (30)$$

where $G_{qp}(k)$ is the channel transfer function between the transmitter of the qth secondary user and the receiver of the pth primary user, and $P_{p,k}^{\text{peak}}$ is the maximum interference over subcarrier k tolerable by the pth primary user. These limits are chosen by each primary user, according to his QoS requirements.

The aim of each secondary user is to maximize his own rate $r_q(\mathbf{p}_q, \mathbf{p}_{-q})$ under the local power constraints in (29) and the additional global interference constraints in (30). The interference constraints however introduce a coupling among the admissible power allocations of all the players, meaning that the secondary users are not allowed to choose their power allocations individually. To keep the optimization as decentralized as possible while imposing global interference constraints, the proposed idea is to introduce a pricing mechanism, controlled by the primary users, through a penalty in the payoff function of each player, so that the best-response power allocation of each secondary user (and thus the interference generated by all the secondary users) will depend on these prices. The challenging goal is then to find the proper decentralized pricing mechanism that guarantees the interference constraints to be satisfied while the secondary users reach an equilibrium. Stated in mathematical terms, we have the following NE problem [49]:

$$(\mathcal{G}_{\text{VI}}): \quad \begin{array}{ll} \underset{\mathbf{p}_q \geq 0}{\text{maximize}} & r_q(\mathbf{p}_q, \mathbf{p}_{-q}) - \sum_{p=1}^{P}\sum_{k=1}^{N} \lambda_{p,k}^{\text{peak}} |G_{qp}(k)|^2 p_q(k) \\ \text{subject to} & \mathbf{p}_q \in \mathcal{P}_q \end{array} \quad (31)$$

for all $q \in \Omega$, where the prices $\lambda_p^{\text{peak}} = \{\lambda_{p,k}^{\text{peak}}\}_{k=1}^{N}$ are chosen such that the following complementary conditions are satisfied:

$$0 \leq \lambda_{p,k}^{\text{peak}} \perp P_{p,k}^{\text{peak}} - \sum_{q=1}^{Q} |G_{qp}(k)|^2 p_q(k) \geq 0,$$
$$\forall p = 1, \ldots, P, \; \forall k = 1, \ldots, N, \quad (32)$$

where the compact notation $0 \leq a \perp b \geq 0$ means $a \cdot b = 0$, $a \geq 0$, and $b \geq 0$. These constraints state that the per-carrier interference constraints must be satisfied together with nonnegative pricing; in addition, they imply that if one constraint is trivially satisfied with strict inequality then the corresponding price should be zero (no punishment is needed in that case).

VI REFORMULATION

Due to the global interference constraints, the coupling among the strategies of the players of $\mathcal{G}_{\text{VI}}$ presents a new challenge for the analysis of this class of Nash games that cannot be addressed using results from game theory or game theoretical models proposed in the literature [15]–[19], [25]. For this purpose, we need the framework given by the more advanced theory of finite-dimensional VIs [26], [27] that provides a satisfactory resolution to the game $\mathcal{G}_{\mathrm{VI}}$, as detailed next. Define the joint admissible strategy set of game $\mathcal{G}_{\mathrm{VI}}$ as

$$\mathcal{K} \triangleq \mathcal{P} \cap \left\{ \mathbf{p} : \sum_{q=1}^{Q} |G_{qp}(k)|^2 \, p_q(k) \leq P_{p,k}^{\mathrm{peak}}, \quad \forall\, p = 1, \ldots, P,\ k = 1, \ldots, N \right\}, \tag{33}$$

with $\mathcal{P} = \mathcal{P}_1 \times \cdots \times \mathcal{P}_Q$, and the vector function $\mathbf{F}(\mathbf{p})$

$$\mathbf{F}(\mathbf{p}) \triangleq \begin{pmatrix} \mathbf{F}_1(\mathbf{p}) \\ \vdots \\ \mathbf{F}_Q(\mathbf{p}) \end{pmatrix}, \ \text{where each } \mathbf{F}_q(\mathbf{p}) \triangleq -\nabla_{\mathbf{p}_q} r_q(\mathbf{p}_q, \mathbf{p}_{-q}) \tag{34}$$

and $\nabla_{\mathbf{p}_q} r_q(\mathbf{p}_q, \mathbf{p}_{-q})$ denotes the gradient of the payoff function $r_q(\mathbf{p}_q, \mathbf{p}_{-q})$ with respect to $\mathbf{p}_q$. It can be shown that solving the game $\mathcal{G}_{\mathrm{VI}}$ is equivalent to solving the VI problem defined by the pair $(\mathcal{K}, \mathbf{F})$, which is to find a vector $\mathbf{p}^* \in \mathcal{K}$ such that [49]

$$(\mathbf{p} - \mathbf{p}^*)^T \mathbf{F}(\mathbf{p}^*) \geq 0, \ \forall\, \mathbf{p} \in \mathcal{K}. \tag{35}$$

Such an equivalence means that if $\mathbf{p}^*$ is a solution of the VI$(\mathcal{K}, \mathbf{F})$, then there exists a set of prices $\boldsymbol{\lambda}^* = (\boldsymbol{\lambda}_p^*)_{p=1}^P \geq \mathbf{0}$ such that $(\mathbf{p}^*, \boldsymbol{\lambda}^*)$ is an equilibrium pair of $\mathcal{G}_{\mathrm{VI}}$; conversely if $(\mathbf{p}^*, \boldsymbol{\lambda}^*)$ is an equilibrium of $\mathcal{G}_{\mathrm{VI}}$, then $\mathbf{p}^*$ is a solution of the VI$(\mathcal{K}, \mathbf{F})$ [49]. Using the above equivalence, we can now study properties of game $\mathcal{G}_{\mathrm{VI}}$ by focusing on the VI equivalent formulation in (35) and using the mathematical framework developed for this theory [26], [27].

EQUILIBRIUM SOLUTIONS OF $\mathcal{G}_{\mathrm{VI}}$: EXISTENCE AND UNIQUENESS

To rewrite the solutions to $\mathcal{G}_{\mathrm{VI}}$ in a convenient form, we introduce, for each q, the waterfilling-like mapping wf_q, defined for any given $\mathbf{p}_{-q} \geq \mathbf{0}$ and $\boldsymbol{\lambda} \geq \mathbf{0}$ as

$$[\mathrm{wf}_q(\mathbf{p}_{-q};\boldsymbol{\lambda})]_k \triangleq \left[\frac{1}{\mu_q + \gamma_q(k;\boldsymbol{\lambda})} - \frac{\sigma_q^2(k) + \sum_{r \neq q} |H_{rq}(k)|^2 p_r(k)}{|H_{qq}(k)|^2} \right]_0^{p_q^{\max}(k)} \tag{36}$$

with $k = 1, \ldots, N$, where $\gamma_q(k;\boldsymbol{\lambda}) = \sum_{p=1}^P |G_{qp}(k)|^2 \lambda_{p,k}^{\mathrm{peak}}$ and $\mu_q \geq 0$ is chosen to satisfy the power constraint $\sum_{k=1}^N [\mathrm{wf}_q(\mathbf{p}_{-q};\boldsymbol{\lambda})]_k \leq P_q$ ($\mu_q = 0$ if the inequality is strictly satisfied).

Existence and uniqueness conditions of a solution to $\mathcal{G}_{\mathrm{VI}}$ follows from those of the VI: Game $\mathcal{G}_{\mathrm{VI}}$ always admits an NE for any set of power and interference constraints, since the VI$(\mathcal{K}, \mathbf{F})$ does (because the set $\mathcal{K}$ is convex and compact and the function $\mathbf{F}(\mathbf{p})$ is continuous in $\mathbf{p}$ on $\mathcal{K}$). Moreover, given the set of the optimal prices $\hat{\boldsymbol{\lambda}} = \{\hat{\boldsymbol{\lambda}}_p^{\mathrm{peak}}\}_{p=1}^P$, the optimal power allocation vector $\mathbf{p}^*(\hat{\boldsymbol{\lambda}}) = (\mathbf{p}_q^*(\hat{\boldsymbol{\lambda}}))_{q=1}^Q$ of the secondary users at the NE of game $\mathcal{G}_{\mathrm{VI}}$ is the solution to the following vector waterfilling-like fixed-point equation:

$$\mathbf{p}_q^*(\hat{\boldsymbol{\lambda}}) = \mathrm{wf}_q(\mathbf{p}_{-q}^*(\hat{\boldsymbol{\lambda}}); \hat{\boldsymbol{\lambda}}), \ \forall q \in \Omega. \tag{37}$$

with wf_q defined in (36).

Invoking uniqueness results of VI [27], we obtain sufficient conditions guaranteeing the uniqueness of the optimal power allocations $\mathbf{p}^*$ of game $\mathcal{G}_{\mathrm{VI}}$. For example, the NE $\mathbf{p}^*$ of $\mathcal{G}_{\mathrm{VI}}$ is unique if the two following set of conditions are satisfied (more general conditions are given in [49]):

Low-received MUI:

$$\sum_{r \neq q} \max_k \left\{ \frac{|H_{rq}(k)|^2}{|H_{rr}(k)|^2} \cdot \mathrm{innr}_{rq}(k) \right\} < 1, \ \forall q \in \Omega, \tag{C5}$$

Low-generated MUI:

$$\sum_{q \neq r} \max_k \left\{ \frac{|H_{rq}(k)|^2}{|H_{rr}(k)|^2} \cdot \mathrm{innr}_{rq}(k) \right\} < 1, \ \forall r \in \Omega, \tag{C6}$$

where the interference-plus-noise to noise ratios $\mathrm{innr}_{rq}(k)$ are defined as

$$\mathrm{innr}_{rq}(k) \triangleq \frac{\sigma_r^2(k) + \sum_t |H_{tr}(k)|^2 p_t^{\max}(k)}{\sigma_q^2(k)}. \tag{38}$$

Conditions (C5)–(C6) have the same nice interpretations of (C1)–(C4): The uniqueness is guaranteed if the interference among the secondary users is not too high, in the sense specified by (C5)–(C6).

DISTRIBUTED ALGORITHMS

To obtain efficient algorithms that distributively compute both the optimal power allocations of the secondary users and prices, we can borrow from the wide literature of solutions methods for VIs [27]. Many alternative algorithms have been proposed in [49] to solve game $\mathcal{G}_{\mathrm{VI}}$ that differ in: i) the signaling among primary and secondary users needed to be implemented, ii) the computational effort; iii) the convergence speed, and iv) the convergence analysis. As an example, we describe two of these algorithms.

The first algorithm we provide is based on the Projection Algorithm with variable steps (for the sake of simplicity, here we use a constant step size) [27, Alg. 12.1.4] and is formally described in Algorithm 2, where the waterfilling mapping wf_q is defined in (36).

ALGORITHM 2: PROJECTION ALGORITHM WITH CONSTANT STEP SIZE

`Data` : $\boldsymbol{\lambda}^{(0)} \geq \mathbf{0}$, and the step size $\tau > 0$.

1: Set $n = 0$;
2: `repeat`
3: Given $\boldsymbol{\lambda}^{(n)}$, compute $\mathbf{p}^*(\boldsymbol{\lambda}^{(n)})$ as the solution to the fixed-point equation
4: $\mathbf{p}_q^*(\boldsymbol{\lambda}^{(n)}) = \mathrm{wf}_q(\mathbf{p}_{-q}^*(\boldsymbol{\lambda}^{(n)}); \boldsymbol{\lambda}^{(n)}), \ \forall q \in \Omega$ (39)
5: Update the price vectors: for all $p = 1, \ldots, P$, and $k = 1, \ldots, N$, compute

6: $\lambda_{p,k}^{(n+1)} = \left[\lambda_{p,k}^{(n)} - \tau\left(P_{p,k}^{\text{peak}} - \sum_{q=1}^{Q}|G_{qp}(k)|^2\, p_q^{*}(k;\lambda^{(n)})\right)\right]^{+}$, (40)

7: `until the prescribed convergence criterion is satisfied.`

The algorithm can be interpreted as follows. In the main loop, at the nth iteration, each primary user p measures the received interference generated by the secondary users and, locally and independently from the other primary users, adjusts his own set of prices $\lambda_p^{(n)}$ accordingly, via a simple projection scheme [see (40)]. The primary users broadcast their own prices $\lambda_p^{(n)}$'s to the secondary users, who then play the game in (31) keeping fixed the prices to the value $\lambda^{(n)}$. The Nash equilibria of such a game can be reached by the secondary users using any algorithm falling in the class of asynchronous IWFA as described in Algorithm 1 (e.g., simultaneous or sequential) and based on mapping $\text{wf} = (\text{wf}_q)_{q\in\Omega}$ in (37). The interesting result proved in [49] is that, both loops—the outer loop performed by the primary users and the inner loop based on iterative waterfilling-like algorithm performed by the secondary users—are guaranteed to asymptotically converge if, e.g., conditions (C5)–(C6) are satisfied and the step size $\tau > 0$ is chosen arbitrarily but smaller than a prescribed value (given in [27]).

The second algorithm we provide is Algorithm 3 that differs from Algorithm 2 in the fact that there is only a major loop in which the secondary and the primary users update their decisions at the same level either sequentially or simultaneously (in the version described in Algorithm 3, the update is simultaneous). Thus, in Algorithm 3, the primary users adjust their prices as soon as the secondary users complete one iteration of their nonequilibrium allocation updates, rather than waiting for a full equilibrium response, as in Algorithm 2.

ALGORITHM 3: PROXIMAL REGULARIZATION ALGORITHM

`Data`: $\lambda^{(0)} \geq \mathbf{0}$, $\mathbf{p}_q = \mathbf{p}_q^{(0)} \in \mathcal{P}_q$ for all $q = 1,\ldots,Q$, and $\zeta > 0$.

1: Set $n = 0$;

2: `repeat`

3: Given $\lambda^{(n)}$, sequentially for $q = 1,\ldots,Q$, compute $\mathbf{p}_q^{(n+1)}(\lambda^{(n)})$ as

4: $\mathbf{p}_q^{(n+1)}(\lambda^{(n)}) = \text{wf}_q(\mathbf{p}_1^{(n+1)},\ldots,\mathbf{p}_{q-1}^{(n+1)},\mathbf{p}_{q+1}^{(n)},\ldots,\mathbf{p}_Q^{(n)};\lambda^{(n)})$, (41)

5: Update the price vectors: for all $p = 1,\ldots,P$, and $k = 1,\ldots,N$, compute

6: $\lambda_{p,k}^{(n+1)} = \left[\lambda_{p,k}^{(n)} - \zeta^{-1}\left(P_{p,k}^{\text{peak}} - \sum_{q=1}^{Q}|G_{qp}(k)|^2\, p_q^{(n+1)}(k;\lambda^{(n)})\right)\right]^{+}$, (42)

7: `until the prescribed convergence criterion is satisfied.`

Even though the per-carrier and global interference constraints impose a coupling among the feasible power allocation strategies of the secondary users, the equilibrium of game $\mathcal{G}_{\text{VI}}$ can be reached using iterative algorithms that are fairly distributed with minimum signaling from the primary to the secondary users. In fact, the primary users, to update their prices, only need to measure the interference generated by the secondary users, which can be performed locally and independently from the other primary users. Regarding the secondary users [see (36)], once $\gamma_q(k;\lambda)$s are given, the optimal power allocation can be computed locally by each secondary user, since only the measure of the received MUI over the N subcarriers is needed. However, the computation of $\gamma_q(k;\lambda)$s requires a signaling among the primary and secondary users: At each iteration, the primary users have to broadcast the new values of the prices and the secondary users estimate the $\gamma_q(k;\lambda)$s. Note that, under the assumption of channel reciprocity, the computation of each term $|G_{qp}(k)|^2\,\lambda_{p,k}^{\text{peak}}$ in $\gamma_q(k;\lambda)$ does not requires the estimate from each secondary user of the (cross-)channel transfer functions between his transmitter and the primary receivers.

CONSERVATIVE IWFA VERSUS FLEXIBLE IWFA

As a numerical example, we compare three different approaches, namely the VI-based formulation (Algorithms 2 and 3), the classical IWFA [14], [24], and the IWFA with individual interference constraints (i.e., a special case of Algorithm 1 applied to game $\mathcal{G}_{\text{soft}}$) [17]–[18], in terms of interference generated at the primary user receivers and the achievable sum-rate from the secondary users; we refer to these algorithms as flexible IWFA, classical IWFA, and conservative IWFA, respectively. As an example, we consider a CR system composed of six secondary links randomly distributed within a hexagonal cell and one primary user (the BS at the center of the cell). The primary user imposes a constraint on the maximum interference that can tolerate. For simplicity in our description, we assume that the primary user imposes a constant interference threshold over the whole spectrum, i.e., $P_{p,k}^{\text{peak}} = 0.01$ for all $k = 1,\ldots,N$ [see (30)]. The individual interference constraints used in the conservative IWFA are chosen so that all the secondary users generate the same interference level at the primary receiver and the aggregate interference satisfies the imposed interference threshold. In Figure 6(a), we plot the power spectral density (PSD) of the interference generated by the secondary users at the receiver of the primary user, obtained, for a given channel realization, and distribution of the secondary users, using the flexible IWFA and the conservative IWFA. As benchmark, we also include the PSD of the interference generated by the classical IWFA [14], [24]. We clearly see from the picture that while classical IWFA violates the interference constraints, both conservative and flexible IWFAs satisfy them but the global interference constraints impose less stringent conditions on the transmit power of the secondary users that those imposed by the individual interference constraints based on the spectral masks. However, this comes at the price of some signaling from the primary to the secondary users.

Thanks to less stringent constraints on the transmit powers of the secondary users, the flexible IWFA is expected to exhibit a much better performance than the conservative IWFA also in terms of rates achievable by the secondary user. Figure 6(b) confirms this intuition, where we plot the average sum-rate of the secondary users achievable by the conservative IWFA and the flexible IWFA as a function of the maximum tolerable interference at the primary receiver, within the same setup described above. The curves are averaged over 500 random i.i.d. Gaussian channel realizations.

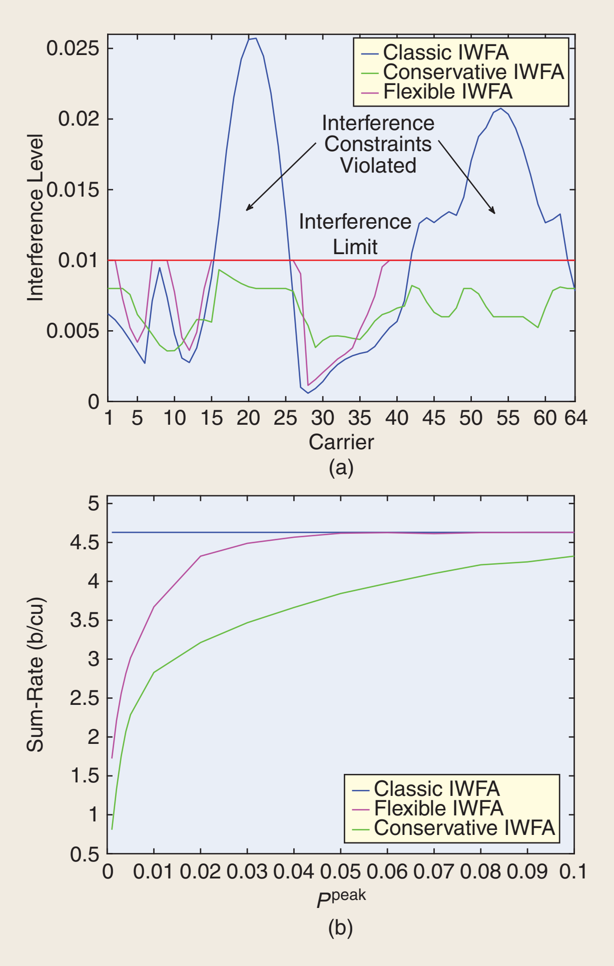
CONVERGENCE SPEED

In Figure 7(a), we plot the worst-case violation of the interference constraint achieved by Algorithms 2 and 3 versus the number of iterations of the outer loop, for a CR system as in Figure 6, composed now of 15 active secondary links. Interestingly, for the example considered in the figure, both Algorithms 2 and 3 experience the same convergence behavior (provided that the step-size is properly chosen) and converge reasonably fast. Thus, in such a scenario, Algorithm 3 is preferred to Algorithm 2, since it requires less iterations among the secondary users. Finally, in Figure 7(b), we compare the performance in terms of convergence speed of the sequential and simultaneous IWFA based on the waterfilling best response with pricing defined in (36), for a given price tuple λ and channel realization. These algorithms are used to compute the (unique) solution of the fixed-point equation (39) in the inner loop of Algorithm 2. In the figure, we plot the rate evolution of the secondary users' links corresponding to the two cited algorithms as a function of the iteration index. To make the figure not excessively overcrowded, we plot only the curves of three out of 15 links. As expected, the sequential IWFA is slower than the simultaneous IWFA, especially if the number of active secondary users is large, since each user is forced to wait for all the users scheduled in advance, before updating his own power allocation. The same qualitative behavior has been observed for different channel realizations and value of prices. The fast convergence behavior of the IWFAs in the inner loop provides an intuitive explanation of why Algorithms 2 and 3 have been experienced to have almost the same convergence speed [Figure 7(a)], provided that the step-size is properly chosen: After the first round of the IWFA in the inner loop, the secondary users are expected to be quite close the NE of the inner loop game already.

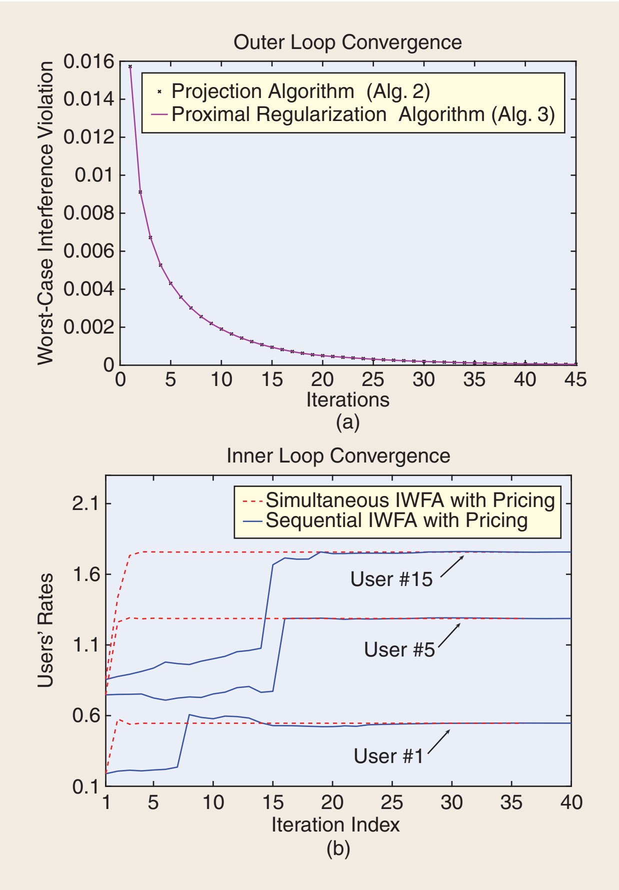
CONCLUSIONS AND FUTURE DIRECTIONS

In this article, we have proposed different NE models to formulate and solve in a distributed way resource allocation problems in CR systems. We have seen how noncooperative game theory and the more general VI theory provide the natural framework to address and solve some of the challenging issues in CR, namely: 1) the establishment of conditions guaranteeing that the dynamical interaction among cognitive nodes, under different constraints on the transmit spectral mask and on interference induced to primary users, admits a (possibly unique) equilibrium; and 2) the design of decentralized algorithms able to reach the equilibrium points, with minimal coordination among the nodes. The proposed algorithms differ in the trade-off between performance (in terms of information rate) achievable by the secondary users and the degree of information to be exchanged between the primary and the secondary users. Thus, they are valid candidates in the two main paradigms of CR systems, namely the common model and the spectral leasing approach. Motivated by the need of low-complexity distributed algorithms for CR applications, we restricted ourselves to discuss some fundamental aspects of noncooperative games, leaving the large area of cooperative game theory out of the scope of this article. Tutorial papers on cooperative game theory based on Nash bargaining problem and coalition games applied to communication systems are [11], [22], and [50], respectively.

The NE points derived in this article were dictated by the need of finding totally decentralized algorithms with minimal coordination among the nodes. However, the NE points may not be Pareto efficient. This raises the issue of how to move from the NE towards the Pareto-optimal tradeoff surface, still using a decentralized approach. Many directions can be exploited to improve the utility region, such as noncooperative games incorporating pricing mechanisms. Prices are indeed introduced as an instrument to induce a distributed incentive for the players to reach more socially efficient NE points. Toward this goal, many heuristics have been proposed



[FIG6] Comparison of IWFA algorithms: Classical IWFA, conservative IWFA, and flexible IWFA. (a) PSD of the interference profile at the primary user's receiver. (b) Achievable average sum-rate versus the interference constraint.



[FIG7] **Example of convergence speed. (a) Worst-case violation of the interference constraints achieved by Algorithms 2 and 3 (flexible IWFA). (b) Secondary users' rates versus the iteration index achieved in the inner loop of Algorithm 2 by the sequential and the simultaneous IWFA with pricing.**

in the literature (see, e.g., [49] and [51]–[52] and references therein), but a formal study of the impact of the prices on the performance of the corresponding NE point remains, to-date, a formidable open problem. Other game theoretical models worth of investigating are those coming from dynamic games [53], for example, in the form of repeated games, where the players learn from their past choices. An early application of repeated games to the spectrum sharing problem over unlicensed bands is [24]. However, the (nontrivial) extension to hierarchical CR networks is, to-date, missing One further direction worth of being exploited to improve the utility region is to consider some refinements of the NE concept as, e.g., in [36]–[37].

In this article, we assumed perfect channel state information and perfect sensing from the secondary users. An interesting extension of the presented approach consists in taking into account the effects of channel and spectrum sensing errors and developing robust transmission strategies, based on either deterministic or random access schemes. This is particularly relevant in CR systems because the strategy adopted by the cognitive users may be more or less aggressive depending on the reliability of their channel/spectrum sensing.

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