Retrieving Dispersion Diagrams of Waveguide-Coupled Planar Photonic-Crystal-Embedded Microresonators

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ABSTRACT
We report a numerical study on retrieving the dispersion diagrams of waveguide-coupled planar photonic-crystal-embedded microresonators (PCEMs) that comprise a finite-size square photonic crystal lattice of air-holes embedded in a high-index-contrast square microcavity. We extract the projected dispersion diagram of the waveguide-coupled PCEM by means of spatial Fourier transform of the simulated waveguide field patterns over a span of frequencies. We use various coupled waveguide widths in order to span a selected range of wavevectors. The retrieved PCEM dispersion diagram exhibits discrete flat bands near resonance frequencies over a relatively wide scope of wavevectors.

Keywords: photonic-crystal-embedded microresonator, finite-size photonic crystal lattice, projected dispersion diagram.

1. INTRODUCTION
Previously [1], we proposed and numerically studied an alternative planar photonic crystal (PC) microresonator that we referred to as two-dimensional (or planar) photonic-crystal-embedded microresonators (PCEMs). A planar PCEM comprises a finite-size PC lattice of holes embedded in a planar microresonator. For example, a square PCEM comprises a finite-size square PC lattice of holes embedded in a square micropillar resonator. Planar PCEMs partially confine light by total internal reflection at the microcavity sidewalls, and resonances occur at discrete frequencies that satisfy phase matching condition in a finite-size PC. We referred to such resonances as dispersion-guided resonances. It is thus conceivable that lightwave in a PCEM sees the cavity-modified finite-size PC dispersion, with discrete flat bands corresponding to PCEM resonances. In contrast to conventional Bragg-reflection PC microresonators [2, 3], PCEMs employ the allowed PC bands and do not require a band-gap structure. Coupling to planar PCEMs can employ relatively low-loss conventional channel waveguides.

In order to better understand PCEMs, it is of essence to obtain the PCEM dispersion diagram. Although dispersion diagrams of infinite two-dimensional (2-D) PC lattice can be readily calculated, calculating dispersion diagrams of cavity-modified finite-size 2-D PC can be a formidable challenge. Here we report the dispersion diagrams of waveguide-coupled planar PCEMs, using 2-D finite-difference time-domain (FDTD) method to simulate the steady-state field patterns over a scope of frequencies and spatial Fourier analysis of the waveguide field patterns. The extracted dispersion diagrams, projected in the direction of the waveguide axis, display conventional waveguide dispersion at off-resonance frequencies and discrete flat bands with narrow stop bands corresponding to PCEM resonances.

2. DEVICE DESIGN AND ANALYSIS
Figure 1(a) shows the schematic of a channel waveguide-coupled planar PCEM-based notch filter. The PCEM comprises a square PC lattice of air-holes embedded in a planar square microresonator. We choose the lattice period \( a = 0.465 \mu m \), the hole radius \( r = 0.3a \), and the square microresonator size \( D = 7a + 0.4a \). We assume a material refractive index \( n = 3.5 \) to represent silicon-based devices. The singlemode channel waveguide of width \( w \) is laterally coupled to the PCEM along the entire microresonator flat sidewall in the \( \Gamma X \) symmetry direction. Resonances are coupled only when the dispersion-guided lightwave components (red dashed arrows) are wavefront-matched with the input-coupled lightwave upon each round trip. Here we study the TE-polarized mode (electric field in plane) using 2-D FDTD method.

Figure 2a shows the FDTD-simulated transmission and reflection spectrum of the waveguide-coupled PCEM with \( w = 0.325 \mu m \). The spectrum reveals three resonances (labeled as A, B, C) between normalized frequency of 0.18 \( c/a \) and 0.22 \( c/a \) (where \( c \) is light velocity in vacuum). Their Q values are also denoted. We remark that the three resonances fall within the first PC band of the corresponding infinite-size square PC lattice (not shown), which exhibits an upper band-edge frequency of ~ 0.22 \( c/a \) at the \( M \) symmetry point. Figures 1b – 1c show the simulated steady-state time-averaged magnetic-field energy distributions of the waveguide-coupled PCEM at resonance mode A (normalized frequency \( f = 0.21375c/a \)), and at resonance mode B (normalized...
We observe a peak internal field intensity enhancement of \( \sim 8 \) at resonance mode A. Figures 1d, 1e show the time-averaged magnetic-field energy distribution of the waveguide center along the longitudinal \( z \) direction for (d) mode A resonance frequency, and (e) mode B resonance frequency.

We apply two-dimensional spatial Fourier transform (SFT) on the simulated waveguide field patterns in order to extract the PCEM-coupled waveguide dispersion diagram. The Fourier transform readily retrieves the waveguide \( k_z \) component. Figure 2a shows the PCEM-coupled waveguide dispersion diagram reconstructed using SFT. We overlay the dispersion diagram onto the transmission spectrum for comparison.

In the off-resonance frequency regions, the retrieved dispersion curve largely follows that of a single channel waveguide. Near the coupled resonance frequencies, the retrieved dispersion curve exhibits flat bands and stop bands characteristics. Figure 2b shows the zoom-in view of the dispersion diagram near mode A resonance frequency.

3. PCEM DISPERSION

Figure 2. (a) Fourier-transform deduced waveguide dispersion (circles) mapped with the transmission spectrum (red line), and reflection spectrum (black line) of the waveguide-coupled PCEM. The coupled waveguide width is 0.325 \( \mu \)m. (b) Zoom-in view of the dispersion diagram near resonance mode A. Red dashed line illustrates the waveguide dispersion. Black dashed line illustrates the PCEM resonance state.
The phase matching between the waveguide mode and the PCEM mode imposes that the retrieved waveguide mode $k_z$ in the proximity of the resonances nearly match the PCEM projected wavevector component in the $\Gamma X$ symmetry direction. Here we choose various coupled waveguide widths $w$'s in order to construct the PCEM dispersion diagram over a selected range of phase-matched projected wavevectors.

Figure 3 shows the overlay of four retrieved PCEM-coupled waveguide dispersion curves with $w = 0.325$ $\mu$m, 0.35 $\mu$m, 0.375 $\mu$m, and 0.4 $\mu$m. The linear waveguide dispersion curves are fit with solid lines. Only the flat band regions near modes A and B frequencies are shown with symbols. The combined dispersion curves near the PCEM resonances thus provides a useful representation of the PCEM band diagram over an extended span of projected wavevectors, from $k \sim 0.325(2\pi/a)$ to $k \sim 0.525(2\pi/a)$. It is worth emphasizing that the retrieved PCEM dispersion diagram according to our analysis exhibits discrete flat bands at resonance frequencies over a relatively wide range of wavevectors. In contrast, an infinite-size PC lattice exhibits a continuous dispersion curve, in which the flat band region is near the band-edge frequency over a relatively narrow range of wavevectors.

4. CONCLUSIONS

In summary, we retrieved the dispersion diagrams of waveguide-coupled planar square photonic-crystal-embedded microresonators (PCEMs), using spatial Fourier transform of the numerically simulated PCEM-coupled waveguide field patterns with various waveguide widths. The constructed PCEM dispersion diagram reveals discrete flat bands near resonance frequencies. The flat bands span over a relatively wide range of wavevectors. It is worth emphasizing that the flat bands are determined by the PCEM resonances and lie below the infinite-size PC band-edge frequency. Our method thus offers useful insights on the dispersion diagrams of cavity-modified finite-size planar photonic crystals.

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REFERENCES

