Hexagonal micro-pillar cavities: multimode resonances and open-loop resonance linewidth broadening

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ABSTRACT

We report our proof-of-principle experiment and modeling of hexagonal micro-pillar (µ-pillar) cavities. A commercial hexagonal silica fiber (125µm plane-to-plane) was side-coupled perpendicularly with a Gaussian beam, thus the fiber acted as a µ-pillar cavity. We observed multimode resonances with typical Q ≈ 2,500 in the tangential directions that are ≈ 120° to the input-coupling cavity sidewall. The observed free spectral range (FSR) ≈ 4.5 nm is consistent with a six-bounce cavity round-trip length. By using wavefront-matching concept, the observed multimode resonances can be attributed to open-loop trajectories. The multiple wavefront-matched open-loop trajectories of the same ray incident angle can result in resonance linewidth broadening. We employed a k-space representation to calculate the hexagonal cavity normal mode locations.

Keywords: micro-pillar cavities, optical resonances, hexagonal, linewidth broadening, WDM add/drop filters

1. INTRODUCTION

Optical µ-pillar cavities have attracted recent interest for applications in integrated photonics due to their compact size (10 - 100 µm lateral dimensions and ≈ µm height) and high-Q resonances. Light can be partially confined by nearly total internal reflection (TIR) at the µ-pillar resonator sidewall. Optical resonances can be excited only when the cavity round-trip wavefronts are wavefront-matched. The µ-pillar cavity can be laterally or vertically coupled with waveguides, and circular µ-pillar wavelength-division multiplexing (WDM) channel add/drop filters have been demonstrated [1, 2, 3]. However, the main drawback of the laterally coupled circular ring and disk micro-cavities is the short interaction length between the curved cavity sidewall and the straight waveguide sidewall. Such short interaction length imposes a sub-micrometer air-gap spacing for evanescent coupling. In order to improve the lateral coupling length, channel add/drop filters based on race-track shaped resonators [4] and polygonal µ-pillar resonators have been recently proposed [5].

The key advantages of the polygonal µ-pillar cavities are two-fold: (1) the entire flat cavity sidewall allows a longer lateral coupling length, and (2) the same cavity modes can be input and output-coupled along the sidewall. Therefore, a wide air-gap spacing can be tolerated for evanescent coupling between the cavity and the laterally coupled straight waveguides. Recently, multimode resonances in square-shaped µ-pillar cavities were experimentally observed by Gaussian beam coupling [6]. Hexagonal microlasers have been reported [7].
In this paper, we report our recent measurement of optical resonances in the elastic-scattering spectra of hexagonal \( \mu \)-pillar cavities, using Gaussian beam tangentially coupled along the cavity sidewall. The observed multimode resonances can be attributed to wavefront-matched six-bounce round-trip trajectories that need not be closed after each round trip. The family of wavefront-matched open-loop trajectories that are at the same incident angle but different coupling positions can result in resonance linewidth broadening. We employed a k-space representation to calculate the hexagonal cavity resonance locations.

2. HEXAGONAL MICRO-CAVITIES MODELING: WAVEFRONT-MATCHING AND K-SPACE

2.1 Ray optics with wavefront-matching

Figure 1 (a) shows typical hexagonal closed-loop trajectories with an incident angle \( \theta = 60^\circ \) (solid and dashed lines). The trajectories have the same round-trip length of \( 3L \), where \( L \) is the distance between two parallel sidewalls, and thus have the same cavity modes. Trajectories with \( \theta \neq 60^\circ \) do not close upon themselves in each six-bounce round-trip. Figure 1 (b) illustrates a typical six-bounce open-loop trajectory with an incident angle \( \theta \neq 60^\circ \), and a complementary incident angle \( (120^\circ - \theta) \) at the adjacent sidewall.

![六边形微腔模型：波前匹配和k空间](image)

In order to partially confine the six-bounce trajectories by TIR, \( \theta \) needs to satisfy \( \theta_c < \theta < (120^\circ - \theta_c) \), where the critical angle \( \theta_c = \sin^{-1} (1/n) \), and \( n \) is the cavity refractive index contrast. For silica in 1550nm wavelength, \( n \approx 1.44 \) \( (\theta_c \approx 44^\circ) \), and thus \( 44^\circ < \theta < 76^\circ \) for six-bounce TIR trajectories. We only consider six-bounce trajectories because three-, four-, and five-bounce trajectories are not confined by TIR in silica hexagonal cavities.

Here, we review the wavefront-matching concept [6] in the context of hexagonal cavities. Only the round-trip trajectories that have the incident wavefront matched with the round-trip wavefront can excite optical resonances. The wavefront-matched trajectories do not need to be closed upon each round trip, as illustrated in Figure 1 (b). The dashed line that is perpendicular to ray GH and AB represents the wavefront. The ray AB and ray GH are wavefront-matched.

Following the discussion of multimode resonances in square \( \mu \)-pillar cavities in [6], we believe such wavefront-matched open-loop trajectories can result in multimode resonances in hexagonal cavities. The optical path length \( L_h \) for a wavefront-matched six-bounce trajectory (denoted as ABCDEFGH in (b)) is as follows:

\[
L_h = n \cdot \text{Polygon}_{ABCDEFGH} = n \cdot 3L \cdot \sin(\theta + 30^\circ)
\]

We remark that \( L_h \) is independent of the trajectory starting position as long as the open-loop trajectory is wavefront-matched, and consequently \( L_h \) is fixed for each open-loop wavefront-matched round-trip. The FSR can be approximated as follows:
2.2 Open-loop resonance linewidth broadening

However, the number of wavefront-matched open-loop round trips N of a fixed θ depends on the ray starting position. Figure 2 (a) – (i) show an assortment of the trajectories for selected θ and the starting positions. For example in Figure 2 (a), the ray starts at position P on sidewall AB with θ = 56.8°. We denote the coupling (starting) position $x = \frac{BP}{BA} = 0.9$. The ray reaches the sidewall CD (instead of the adjacent sidewall BC) and escape refractively (i.e. N=0) according to Snell’s law because the incident angle at sidewall CD is less than $\theta_c$. We term this as the walk-off condition for the
open-loop trajectories. We consider \( x \) between 0.1 and 0.9 only because practical hexagonal \( \mu \)-pillar cavities can have rounded corners. Only a limited span of \( \theta \) within the TIR confinement can make wavefront-matched round trips before walk-off and the subsequent refractive escape. For all \( \theta < 53.5^\circ \), \( N = 0 \).

Figure 2 (b) shows the open-loop trajectory for \( \theta = 56.8^\circ \) and \( x = 0.5 \). The ray can reach four sidewalls before walk-off and the refractive escape at the input coupling sidewall. Figure 2 (c) illustrates the open-loop trajectory with the same \( \theta \) and \( x = 0.1 \) that has \( N = 1 \) before the ray eventually walks off at the input coupling sidewall. Figure 2 (d), (e), (f) illustrate that for \( \theta = 58.7^\circ \), \( N \) varies from 0 to 3. Figure 2 (g), (h), (i) show that for \( \theta = 59.5^\circ \), \( N \) varies from 1 to 10. \( N \) becomes infinitely large when \( \theta \) approaches 60°.

The different \( N \) allows different cavity lifetime (before refractive escape) for the same \( L_h \) at different \( x \). When the hexagonal cavity is input coupled along the entire cavity sidewall, the same mode will see a distribution of cavity lifetimes, and therefore result in a broadened resonance linewidth. Figure 3 illustrates schematically the open-loop linewidth broadening due to the superposition of different \( Q \)'s (cavity lifetimes) at the same wavelength.

![Figure 3. Schematic of the open-loop resonance linewidth broadening due to the superposition of different \( Q \)'s at the same \( \lambda \).](image)

### 2.3 k-space

The resonance modes of a hexagonal cavity can be represented by the normal modes in \( k \)-space. We assume that the hexagonal cavity is composed of three pairs of Fabry-Perot mirrors (denoted as \( \text{aa'} \), \( \text{bb'} \), and \( \text{cc'} \)) with a plane-to-plane distance \( L \), as illustrated in Figure 4 (a). In the Fabry-Perot cavity \( \text{aa'} \), the \( k \)-vector \( k_a \) can be discretized as \( m_a \pi/L \), where \( m_a \) is an integer (i.e., 0, 1, 2). Likewise, the Fabry-Perot cavities \( \text{bb'} \) and \( \text{cc'} \) have \( k \)-vectors \( k_b \) and \( k_c \) discretized as \( m_b \pi/L \) and \( m_c \pi/L \), respectively, where \( m_b \) and \( m_c \) are integers (i.e., 0, 1, 2).

Using the \((k_x, k_y, k_z)\) bases, we define the orthogonal \( k \)-space coordinates \((k_x, k_y)\) (shown in Figure 4 (a)) as follows:

\[
\begin{align*}
    k_x &= \frac{\sqrt{3}}{2} \cdot (k_b + k_c) = \frac{\sqrt{3}}{2} \cdot \frac{\pi}{L} \cdot (m_b + m_c) = \frac{\sqrt{3}}{2} \cdot \frac{\pi}{L} \cdot m_x \\
    k_y &= k_a + \frac{1}{2} k_b - \frac{1}{2} k_c = \frac{1}{2} \cdot \frac{\pi}{L} \cdot (2m_b - m_c) = \frac{1}{2} \cdot \frac{\pi}{L} \cdot m_y
\end{align*}
\]

(3)

(4)

where \( m_x = (m_b + m_c) \), and \( m_y = (2m_b - m_c) \) are integers. The hexagonal cavity normal mode wavelengths can be calculated as follows:

\[
    k^2 = \left( \frac{2\pi n}{\lambda} \right)^2 = k_x^2 + k_y^2 = \left( \frac{\pi}{2L} \right)^2 \cdot (3m_x^2 + m_y^2)
\]

(5)
where $\lambda$ is the free-space wavelength of the normal mode $(m_x, m_y)$. Here, $\tan \theta = k_x / k_y$, where $\theta$ is the ray incident angle as shown in Figure 1(a). Since $k_x / k_y$ is discretized, only specific trajectories will give rise to normal modes.

Figure 4. (a) Schematic of a hexagonal cavity composed of three pairs of Fabry-Perot mirrors $aa'$, $bb'$ and $cc'$. $k$-vectors $k_a, k_b, k_c$ for each of the Fabry-Perot mirrors are defined. $k$-space coordinates in $k_x$ and $k_y$ is illustrated. (b) Schematic $k$-space representation of a large-sized hexagonal micro-cavity. The discrete modes are denoted as crosses. The wavelength range ($\lambda_{\text{short}}, \lambda_{\text{long}}$) and the TIR confinement angle range [$\tan \theta_c, \tan (120^\circ - \theta_c)$] are emphasized. The $\tan 60^\circ$ line is shown for reference.

Figure 5. (a) Calculated $(m_x, m_y)$ modes of an L=125µm hexagonal micro-cavity ($n=1.44$) with $\theta = [47.2^\circ, 50.9^\circ]$. The $\lambda$ range is $[1530nm, 1560nm]$. Two sets of modes are labeled as A, B, C and A', B' and C'. The dashed lines represent $\tan 48.4^\circ$ and $\tan 49^\circ$. 
Figure 4 (b) shows the k-space schematic in (k_x, k_y) solutions for a large-sized hexagonal micro-cavity for a wavelength range (λ_short, λ_long) and the angle region [θ_c=44º, 120º-θ_c=76º]. All the modes except for θ=60º can be attributed to wavefront-matched six-bounce open-loop trajectories. Hence, large-sized hexagonal cavities are multi-mode.

![k-space schematic](image)

Figure 5 (a) shows the calculated (m_x, m_y) modes between [47.3°, 50.9°] for an L=125µm hexagonal micro-cavity (n=1.44) in 1530nm – 1560nm wavelength range. Each mode (with degeneracy) is denoted as a dot. Figure 5 (b) shows the k-space modes in a θ-spectrum (θ vs. λ plot) in 1545nm – 1555nm range. Two sets of modes labeled as A, B, C and A', B', C' corresponding to θ ≈ 48° - 49.5° are denoted in both Figure 5 (a) and (b). The calculated wavelength spacing between mode A and A' is 4.36nm, 4.41nm for B and B', and 4.39nm for C and C'.

3. FIBER SCATTERING EXPERIMENT

A commercial hexagonal silica fiber was employed in the elastic-scattering experiment to measure hexagonal cavity resonances. The hexagonal fiber (L=125µm) was side-coupled perpendicularly by a Gaussian beam, thus acted as a hexagonal μ-pillar cavity. Figure 6 shows the experimental setup. A Gaussian beam from a wavelength-tunable diode laser (1510nm-1580nm wavelength range, laser linewidth <300 kHz) was weakly focused (by an ≈ f/10 - f/16 lens) tangentially onto the fiber flat sidewall. The estimated beam width was ≈ 30µm - 50µm. The incident polarization was set to be parallel to the fiber axis (TM mode) using a Glan-Taylor polarizer. We tuned the separation between the laser beam and the fiber sidewall in order to excite the cavity modes. The elastic-scattering spectrum was imaged (with an acceptance angle ≈ 4°) from the fiber flat sidewall onto a 62.5µm-core multimode fiber, and then collected by an InGaAs photodiode. A Glan-Taylor analyzer was placed after the imaging lens to measure the TM spectrum. The spectral resolution was ≈ 0.01 nm. The inset shows the top-view of the fiber. The hexagonal fiber plane-to-plane distance was ≈ 125µm ± 4µm. The fiber corners were rounded.
Figure 6. Schematic of the experimental setup. The inset shows the top-view of the hexagonal fiber.

Figure 7. Measured TM-polarized scattering spectrum (1545nm - 1560nm) imaged at \( \approx 120^\circ \) from the laser beam direction. Four sets of modes are labeled as A, B, C, D. The measured FSR is \( \approx 4.5\text{nm} \). The inset shows a schematic of the Gaussian beam (the thick arrow) grazing the hexagonal fiber sidewall and the output was collected at 120° from the Gaussian beam direction (the dashed arrow).
Figure 7 shows the measured TM-polarized elastic-scattering spectrum (1545nm – 1560nm) imaged at ≈ 120˚ ±2˚ from the laser beam direction. The inset shows a schematic of the Gaussian beam coupling experiment. Multimode resonances (denoted as A, B, C, and D) of typical Q ≈ 2,500 were observed. Mode order B has the maximum observed Q ≈ 15,000. The Q’s are expected to be reduced by the open-loop resonance linewidth broadening. The measured FSR is ≈ 4.5nm, consistent with the six-bounce round-trip length (using Eqs. (1) and (2), FSR ≈ 4.44nm).

The measured FSR suggests (m_x, m_y) modes near θ ≈ 49˚ with L = 125µm, as shown in Figure 5 (b). However, according to ray tracing with wavefront matching, we expect N = 0 when θ = 49˚, and thus no resonance should be excited. We remark that when θ > 53.5˚, N becomes non-zero, yet the calculated FSR according to k-space is only (3.32nm-3.92nm). We are currently working on further measurement and modeling to resolve this apparent dilemma.

4. CONCLUSION

In summary, we have experimentally investigated and modeled the optical resonances of hexagonal μ-pillar cavities. Multimode resonances were observed in the elastic-scattering spectra of a 125µm-sized hexagonal silica fiber. The observed FSR is consistent with a six-bounce cavity round-trip length. By using wavefront-matching concept, the observed multimode resonances can be attributed to open-loop round-trip trajectories. The ray-tracing suggests that only when θ approaches 60˚ can large number of wavefront-matched round-trip trajectories occur. The superposition of multiple wavefront-matched open-loop trajectories of the same ray incident angle but different coupling position can result in resonance linewidth broadening. A k-space modeling was employed to calculate the hexagonal cavity normal mode locations. However, the comparison between the measured FSR with the calculated FSR according to the k-space model suggests that the observed modes have θ ≈ 49˚ which do not constitute wavefront-matched trajectories.

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